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A high order special relativistic hydrodynamic and magnetohydrodynamic code with space–time adaptive mesh refinement

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ABSTRACT

We present a high order one-step ADER–WENO finite volume scheme with space–time adaptive mesh refinement (AMR) for the solution of the special relativistic hydrodynamic and magnetohydrodynamic equations. By adopting a local discontinuous Galerkin predictor method, a high order one-step time discretization is obtained, with no need for Runge–Kutta sub-steps. This turns out to be particularly advantageous in combination with space–time adaptive mesh refinement, which has been implemented following a "cell-by-cell" approach. As in existing second order AMR methods, also the present higher order AMR algorithm features time-accurate local time stepping (LTS), where grids on different spatial refinement levels are allowed to use different time steps.

We also compare two different Riemann solvers for the computation of the numerical fluxes at the cell interfaces. The new scheme has been validated over a sample of numerical test problems in one, two and three spatial dimensions, exploring its ability in resolving the propagation of relativistic hydrodynamical and magnetohydrodynamical waves in different physical regimes. The astrophysical relevance of the new code for the study of the Richtmyer–Meshkov instability is briefly discussed in view of future applications.

1. Introduction

The numerical modeling of complex astrophysical flows that involve relativistic processes requires the development of more and more sophisticated codes. Relevant examples of relativistic phenomena whose understanding can greatly benefit from hydrodynamic and magnetohydrodynamic numerical simulations include extragalactic jets, gamma-ray-bursts, accretion onto compact objects, binary mergers of neutron stars (or black holes), relativistic heavy-ion collisions and so on.

Until few years ago, most of the applications in numerical relativistic hydrodynamic (RHD) and magnetohydrodynamic (RMHD) used second-order accurate, typically TVD, numerical codes. The scientific progress that has been made possible by these implementations is rather significant and not always appreciated enough, see the living reviews by Martí and Müller [1] and Font [2] plus references therein. However, the necessity of improving the accuracy of the computations, especially in the presence of complex phenomena such as instabilities or turbulence, combined with computational resources which are inevitably always limited, has motivated a strong research effort along two different directions. The first direction is represented by the development of high order schemes (better than second order in space and time), while the second direction consists in the implementation of efficient adaptive mesh refinement (AMR) algorithms. Taken separately, high order numerical schemes and AMR techniques have a long history, which is embarrassing to summarize in few words. The first high order special relativistic numerical scheme is due to Dolezal [3], who, in the context of ultra-relativistic nuclear collision experiments, implemented a conservative finite difference scheme using ENO reconstruction in space and Runge–Kutta time integration, but without Riemann solvers. The first transposition of this approach to the astrophysical context is due to Del Zanna and Bucciantini [4], who, in addition, used local Riemann problems to guarantee the upwind character of the scheme. Since then, several high order schemes have been proposed and applied to a variety of different astrophysical problems, with and without magnetic fields, both in the special and in the general relativistic regime (see [5–10]). Though different under many respects, a common feature

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of all these approaches is the use of a multi-step Runge–Kutta time integrator. A few years ago, Dumbser et al. [11] proposed an alternative idea for obtaining a high order integration in time, that avoids Runge–Kutta schemes altogether and originates from the ADER philosophy of Toro and Titarev [12]. According to this idea, which is followed also in this work, an arbitrary high order numerical scheme with just one step for the time update can be obtained, provided a high order time evolution is performed locally (namely within each cell), for the reconstructed polynomials. The first implementation of such ADER schemes in the context of ideal relativistic magnetohydrodynamics can be found in [13] and has later been successfully extended also to the non-ideal relativistic MHD equations in [14].

The implementation of AMR techniques has also a rich tradition in relativistic hydrodynamics and magnetohydrodynamics.[1] The first occurrence is documented in [16] followed by Dünmez [17], Evans et al. [18], van der Holst et al. [19], Tao et al. [20], Etienne et al. [21], De Colle et al. [22], Liebling et al. [23], Lehner et al. [24] and East et al. [25].

The combination of high order relativistic codes with AMR has a much more recent history. Relevant examples are given by the works of Mignone et al. [26], who combined the AMR library CHOMBO, which originated from the original work of Berger and Oliger [27], Berger [28] and Berger and Colella [29], with the versatile PLUTO code, using a Corner-Transport-Upwind scheme together with a third-order WENO reconstruction in space; Anderson et al. [30], who solved the equations of general relativistic magnetohydrodynamics using a conservative finite difference scheme (with reconstruction of primitive variables plus Riemann solvers); Zhang and MacFadyen [31], who implemented both a conservative finite difference scheme (with reconstruction of fluxes and no need for Riemann solvers) and a finite volume method, within the block-structured AMR package PARAMESH of MacNeice et al. [32].

Contrary to the above mentioned approaches, all of them sharing a TVD Runge–Kutta for the time integration, in this paper we present an ADER–WENO finite volume scheme for solving the special relativistic magnetohydrodynamics equations, with adaptive mesh refinement. In [33] we have proposed the first ADER-AMR finite volume numerical scheme for the Newtonian Euler equations and here we propose the relativistic extension of our new approach. The use of a one-step scheme in time allows the implementation of time-accurate local time stepping (LTS) in a very natural and straight forward manner and has already been successfully applied in the context of high order Discontinuous Galerkin schemes with LTS (see [34–37]).

The outline of this paper is the following. In Section 2 we briefly recall the conservative formulation of special relativistic hydrodynamics. Section 3 is devoted to the description of the numerical method, while Section 4 contains the results of the new scheme. Finally, Section 5 concludes our work, with a discussion about future astrophysical applications. In the following we will assume a signature $\{-,+,+,+\}$ for the space–time metric and we will use Greek letters $\mu,\nu,\lambda,\ldots$ (running from 0 to 3) for four-dimensional space–time tensor components, while Latin letters $i,j,k,\ldots$ (running from 1 to 3) will be employed for three-dimensional spatial tensor components. Moreover, we set the speed of light $c=1$ and we adopt the Lorentz–Heaviside notation for the electromagnetic quantities, such that all $\sqrt{4\pi}$ factors disappears.

[1] We are not mentioning here the whole family of AMR implementations in vacuum space–times, namely without matter, which were initiated by Choptuik [15].

2. Special relativistic magnetohydrodynamics

In the following we consider a perfect magneto-fluid, under the assumption of infinite conductivity (ideal RMHD), in a Minkowski space–time with Cartesian coordinates, for which the metric is given by

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + dx^2 + dy^2 + dz^2. \tag{1}$$

The fluid is described by an energy–momentum tensor $T^{\alpha\beta}$

$$T^{\alpha\beta} = (\rho h + b^2)u^\alpha u^\beta + (p + b^2/2)g^{\alpha\beta} - b^\alpha b^\beta, \tag{2}$$

where u^α is the four-velocity of the fluid, b^α is the four-vector of the magnetic field, $b^2 = b_\alpha b^\alpha$, while ρ, $h = 1+\epsilon+p/\rho$, ϵ and p are the rest-mass density, the specific enthalpy, the specific internal energy, and the thermal pressure, respectively. All these quantities are measured in the co-moving frame of the fluid. We assume the pressure is related to ρ and ϵ through the ideal-gas equation of state (EOS), i.e.

$$p = \rho\epsilon(\gamma - 1), \tag{3}$$

where γ is the (constant) adiabatic index of the gas. The equations of special relativistic magneto-hydrodynamics, can be written in covariant form simply as

$$\nabla_\alpha(\rho u^\alpha) = 0, \tag{4}$$

$$\nabla_\alpha T^{\alpha\beta} = 0, \tag{5}$$

$$\nabla_\alpha F^{*\alpha\beta} = 0, \tag{6}$$

where $F^{*\alpha\beta}$ is the dual of the electromagnetic tensor [38]. However, for numerical purposes it is convenient to recast them in conservative form as [39–41]

$$\partial_t \mathbf{u} + \partial_i \mathbf{f}^i = 0, \tag{7}$$

where the conserved variables and the corresponding fluxes in the i direction are given by

$$\mathbf{u} = \begin{bmatrix} D \\ S_j \\ U \\ B^j \end{bmatrix}, \qquad \mathbf{f}^i = \begin{bmatrix} v^i D \\ W^i_j \\ S^i \\ \epsilon^{jik}E^k \end{bmatrix}. \tag{8}$$

The conserved variables (D, S_j, U, B_j) are related to the rest-mass density ρ, to the thermal pressure p, to the fluid velocity v_i and to the magnetic field B_i by[2]

$$D = \rho W, \tag{9}$$

$$S_i = \rho h W^2 v_i + \epsilon_{ijk}E_j B_k, \tag{10}$$

$$U = \rho h W^2 - p + \frac{1}{2}(E^2 + B^2), \tag{11}$$

where ϵ_{ijk} is the Levi-Civita tensor and δ_{ij} is the Kronecker symbol. We have used $W = (1-v^2)^{-1/2}$ to denote the Lorentz factor of the fluid with respect to the Eulerian observer at rest in the Cartesian coordinate system, while E_i are the components of the electric field, which, in ideal magnetohydrodynamics, is simply given by $\vec{E} = -\vec{v} \times \vec{B}$. The tensor

$$W_{ij} \equiv \rho h W^2 v_i v_j - E_i E_j - B_i B_j + \left[p + \frac{1}{2}(E^2 + B^2)\right]\delta_{ij} \tag{12}$$

[2] We note that, since the space–time is flat and we are using Cartesian coordinates, the covariant and the contravariant components of spatial vectors can be used interchangeably, namely $A_i = A^i$, for the generic vector $\vec{A}$.