Reliability Engineering and System Safety 157 (2017) 1–12

Contents lists available at ScienceDirect

Reliability Engineering and System Safety

journal homepage: www.elsevier.com/locate/ress

Hazard tolerance of spatially distributed complex networks

Sarah Dunn*, Sean Wilkinson

Newcastle University, Newcastle, UK

ARTICLE INFO

Article history:
Received 12 February 2016
Received in revised form
4 August 2016
Accepted 15 August 2016
Available online 18 August 2016

Keywords:
Network theory
Spatial networks
Resilience
Reliability
Infrastructure systems
Spatial hazard

ABSTRACT

In this paper, we present a new methodology for quantifying the reliability of complex systems, using techniques from network graph theory. In recent years, network theory has been applied to many areas of research and has allowed us to gain insight into the behaviour of real systems that would otherwise be difficult or impossible to analyse, for example increasingly complex infrastructure systems. Although this work has made great advances in understanding complex systems, the vast majority of these studies only consider a systems topological reliability and largely ignore their spatial component. It has been shown that the omission of this spatial component can have potentially devastating consequences. In this paper, we propose a number of algorithms for generating a range of synthetic spatial networks with different topological and spatial characteristics and identify real-world networks that share the same characteristics. We assess the influence of nodal location and the spatial distribution of highly connected nodes on hazard tolerance by comparing our generic networks to benchmark networks. We discuss the relevance of these findings for real world networks and show that the combination of topological and spatial configurations renders many real world networks vulnerable to certain spatial hazards.

1. Introduction

Infrastructure systems, including water, electricity, transportation and telecommunication, are of critical importance to our modern communities. The reliability of these physical assets and the services they provide are vital for ensuring national security, public health and productivity [20]. It is therefore no surprise that the reliability of these systems has received a great deal of attention in recent years [23]. However, these systems are becoming increasingly complex and interdependent, meaning that they now rely on each other to function normally [15,21], and this increased complexity and reliance is making these networked infrastructure systems harder to manage and assess [33]. We therefore require new tools and techniques to assess their reliability. One possible solution is to use a network graph theory approach to quantify the reliability of these complex infrastructure systems.

Network graph theory has previously been used to analyse a range of systems and provides a rigorous mathematical basis for the analysis of connected elements, enabling aspects of aggregate performance of networked systems to be rapidly calculated [10]. Network models are being increasingly used to improve our understanding of: social systems [2,26,3], neural networks [35,36,6], biological networks [34] and computer science systems [37], amongst others. Studies applying network theory to real-world systems, have recently turned from the analysis of social and biological networks, where space is not traditionally a governing factor, to the analysis of infrastructure systems, which can be distributed over vast geographic regions [14,16,28]. In the case of infrastructure systems, it has been assumed that because many of these networks have been shown to be topologically resilient to a random hazard (e.g. a reliability failure of individual components) they are also resilient to spatially dispersed random hazards (e.g. snowstorm, windstorm). However, Wilkinson et al. [38] analysed the impacts of the Eyjafjallajökull volcano to the European Air Traffic Network and found that this network showed a surprising vulnerability to this hazard, contradicting its assumed topological hazard tolerance. They found that this vulnerability was due to a combination of its topological characteristics and geographical distribution of airports in the network, which is not accounted for in traditional network theory studies, which only consider network topology. The little research that has analysed real-world spatial networks (e.g. infrastructure systems) focuses mainly on characterising the topology of the system, while the spatial element of the same network receives less attention - if not neglected entirely [5]. There are a few studies that have considered the “spatial” resilience of interdependent gas and electrical networks [29,31] or the resilience of China air traffic network [24,30], for example. However, all of these studies have assessed specific real-world applications of spatial resilience and have not considered the overarching, or inherent, resilience of spatial networks in general.

In this paper, we aim to give an assessment of the spatial

* Corresponding author.
E-mail address: sarah.dunn@ncl.ac.uk (S. Dunn).

http://dx.doi.org/10.1016/j.ress.2016.08.010
0951-8320/

hazard tolerance of a range of complex networks, in a similar manner to the study by Albert et al. [1] who considered topological resilience. To achieve this, we provide a robust framework that can quantify the reliability of a complex system to a range of spatial hazards. Unlike previous spatial hazard studies, we do no focus solely on one real-world system but instead generate a range of synthetic networks (termed 'benchmark networks') to use in our resiliency testing. However, we do show that these benchmark networks are characteristic of real-world systems and relate our findings from the 'benchmark networks' to these real-world systems. We consider three classes of relational network model (random, scale-free and exponential networks) which are combined with two different spatial nodal configurations and assess their hazard tolerance to two different locations of a 'growing' spatial hazard.

The rest of the paper is structured as follows: in Section 2, the paper considers the spatial nodal configuration of the 'benchmark networks' and Section 3 considers their network class (i.e. topology). Section 4 develops the spatial hazard models to be used to in our analysis, to which our 'benchmark networks' are then subjected in Section 5. Finally, Section 6 provides conclusions and ideas for future research.

2. Nodal configuration

There has been very little research into the nodal locations of real-world networks. The majority of previous research has investigated pre-existing real-world networks and has therefore used the actual nodal locations [17], or has used purely topological models [17,18,7]. The location of nodes within a real-world network is a very complex problem. In the case of real world systems, nodes may represent cities, regions within cities or individual infrastructure components. Furthermore these systems are dynamic, evolving over time in response to a myriad of drivers such as demographic shifts, technological advancement and availability of resources. Therefore, we generate a range of generic nodal locations and use these to form spatial 'benchmark networks' for testing resiliency. These benchmarks capture the overall distribution of nodes in geographical networks, but not necessarily the small scale local areas of high density nodes (e.g. the model will capture the spread of airports over a continent, but not necessarily the high density of airports clustered around a population centre). This allows us to draw conclusions on the overall hazard tolerance of spatial networks, in a similar manner to traditional topological hazard assessments.

In this paper, we simulate two different spatial nodal layouts and contain them within a 'spatial boundary', outside which no nodes are allowed to form. In the case of a real-world network, this spatial boundary could represent the extent of a land boundary or air space in the case of an air traffic network. We are considering generic spatial layouts in this paper, rather than simulating one area in particular, and have therefore chosen to enclose the networks in a circular boundary. The two different spatial nodal layouts used in this paper are:

- *Uniform with distance* (Fig. 1(a)) – the number of nodes increases linearly with distance away from the geographic centre of the network
- *Uniform with area* (Fig. 1(b)) – the nodes are spread evenly over the network.

The spatial distributions for the two nodal layouts are shown in Fig. 1. These distributions plot the number of nodes against distance from the geographic centre. From this figure, it can be seen that the uniform with distance configuration shows a linear relationship between the proportion of nodes and the distance from geographical centre, whereas the uniform with area configuration exhibits a quadratic relationship.

3. Network classes and models

Relational network models do not include a spatial component, therefore we modify the traditional generation algorithms of random, scale-free and exponential networks to generate a range of spatial networks. All of the generated networks used in this paper have 500 nodes and approximately 3200 links. We have chosen to generate scale-free and exponential networks as they have been shown to capture the real-world characteristics of many infrastructure systems (for example: [32,39]). Whilst, random networks are often used in tests of network robustness to determine if a more structured network is resilient or vulnerable to the applied hazard, due to the homogeneous nature [22,25].

3.1. Random network

In this paper, we generate the random networks using the algorithm of Erdos and Renyi [13]. In this generation algorithm, each pair of nodes is considered in turn and a connection (link) is made between them based upon a value of linking probability (the higher this value the more likely it is that a link will be generated). If the linking probability is equal to 1, then the network will be saturated (i.e. it will have the maximum possible number of links) and if this value equals 0 there will be no links in the network. We do not modify this generation algorithm to take into account the spatial distance between nodes as we are not seeking to create the most efficient network possible (i.e. we are not seeking to minimise, or maximise, the distances between pairs of nodes). In this paper, we are using the random network as a benchmark for tests of resilience for the other two more sophisticated network classes and therefore choose the linking probability to result in approximately the same number of links as these two networks (around 0.025, which results in a network with approximately 3200 links). The degree distributions for the generated networks can be seen in Fig. 2(a) and the associated spatial degree distributions are shown in Fig. 2(b).

3.2. Scale-free network

The scale-free network was first identified and developed by Barabasi and Albert [4] and is based upon the ideas of *growth* and *preferential attachment* [5]. These networks are formed by starting with an initial number of isolated nodes, m_0, which is usually a small percentage of the total number of nodes in the network. New nodes are then added to the network at each 'time step' (i.e. 'growing' the network) until the total number of nodes in the network is reached. These added nodes have between 1 and m_0 links attached to them and connect to the existing nodes in the network based upon the idea of 'preferential attachment'. The probability of attaching to each existing node is calculated based upon its degree, with the nodes with a high degree being more likely to 'attract' a link from the new node (i.e. the rich get richer). It is this 'preferential attachment' rule which results in a few high degree nodes and many small degree nodes in the network. If a spatial layout of the network as nodes are introduced into the network those nodes that are introduced early in the process have more chance to 'attract' links from other nodes compared to nodes introduced later to the network and are therefore more likely to have a higher degree than those introduce late, which in turn has a significant impact upon their spatial hazard tolerance [11].

Therefore we study three methods of choosing the introduction