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Estimation of rare event probabilities in power transmission networks subject to cascading failures

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ABSTRACT

Cascading failures seriously threat the reliability/availability of power transmission networks. In fact, although rare, their consequences may be catastrophic, including large-scale blackouts affecting the economics and the social safety of entire regions. In this context, the quantification of the probability of occurrence of these events, as a consequence of the operating and environmental uncertain conditions, represents a fundamental task. To this aim, the classical simulation-based Monte Carlo (MC) approaches may be impractical, due to the fact that (i) power networks typically have very large reliabilities, so that cascading failures are rare events and (ii) very large computational expenses are required for the resolution of the cascading failure dynamics of real grids. In this work we originally propose to resort to two MC variance reduction techniques based on metamodeling for a fast approximation of the probability of occurrence of cascading failures leading to power losses. A new algorithm for properly initializing the variance reduction methods is also proposed, which is based on a smart Latin Hypercube search of the events of interest in the space of the uncertain inputs. The combined methods are demonstrated with reference to the realistic case study of a modified RTS 96 power transmission network of literature.

1. Introduction

In recent years, power outages and interruptions have been occurring in many countries, with large consequences. For example, the major of Northeast America in 2003 caused a 6 billion dollars economic loss for the region [1,2] and several other social consequences of power interruptions, e.g. related to transportation, food storage and credit card operations, just to mention a few of them [3].

Blackouts are the outcomes of cascades of failures, initiated, in turn, by the failures of a limited set of components, due, for example, to overloads generated by excessive load demands, loss of generation, human errors in network operation, or to external events, e.g. caused by extreme environmental conditions, such as lightning, icing, floods, wind storms, earthquakes, etc. Subsequently, other components fail and are disconnected to avoid further severe damage.

Traditionally, a power transmission network is designed and operated so that a single component disconnection cannot give rise to cascading failures ($N-1$ criterion [4]); however, rare combinations of circumstances, uncommon events or inadequate countermeasures may result in further line disconnections, eventually leading to failure propagation. Extremely severe natural events may even directly fail the components of the network.

In this work, we propose to evaluate the reliability of a power transmission network operating under uncertain environmental conditions. Quantitatively, the problem amounts to computing the probability that an initial, limited outage yields a cascading failure with final load shedding larger than zero (or any other predefined threshold).

Mathematically, the problem can be framed as follows. We consider the model $\mathcal{G}$ of the system response Y to the vector of uncertain inputs $\boldsymbol{x}$:

$$Y=\mathcal{G}(x) \tag{1}$$

where $\boldsymbol{x}$ is a n-dimensional random vector $\boldsymbol{x}=\{x_1, x_2, \ldots, x_n\}$, with probability density function (pdf) $f(\boldsymbol{x})$. The model $\mathcal{G}(\boldsymbol{x})$ is often called the system performance function. The system failure is usually defined as the event $\{\mathcal{G}(\boldsymbol{x})>0\}$, where the set of values $\boldsymbol{x}:\mathcal{G}(\boldsymbol{x})=0$ is defined as limit state and $\boldsymbol{x}:\mathcal{G}(\boldsymbol{x})>0$ is called failure domain. Then, the system failure probability is:

$$P_f=P\left[\mathcal{G}(x)>0\right] \tag{2}$$

In the present work, the performance function $\mathcal{G}(\boldsymbol{x})$ is given by the combination of the network line failure model and the cascading failure

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model, whose output is the final load shedding, as it will further detailed in Section 4. Correspondingly, the failure probability P_f is the probability that an outage yields a cascading failure event leading to a final load shedding larger than zero.

Simulation-based methods, i.e. Monte Carlo (MC) computational schemes, are the most widely used for estimating the probability of failure P_f. In the crude MC scheme, a large number (N_{MC}) of input vector values $\boldsymbol{x}$ is sampled from the joint pdf $f(\boldsymbol{x})$ and the performance function $\mathcal{G}(\boldsymbol{x})$ is evaluated in correspondence of the available N_{MC} input points. A failure indicator variable is defined as:

$$I_F\left(x_i\right) = \begin{Bmatrix} 1 & if\,\mathcal{G}(x_i) > 0 \\ 0 & if\,\mathcal{G}(x_i) \le 0 \end{Bmatrix} \quad (3)$$

The sample mean of the values of the indicator variable obtained is the MC-based estimator of the failure probability:

$$P_f \approx \hat{P}_f = \frac{1}{N_{MC}} \sum_{i=1}^{N_{MC}} I_F(x_i) \quad (4)$$

The accuracy of the estimates can be expressed in terms of the coefficient of variation (δ), defined as the ratio of the sample standard deviation and $\hat{P}_f$ [1]:

$$\hat{\delta}_{MC} = \sqrt{\frac{1-\hat{P}_f}{\hat{P}_f N_{MC}}} \quad (5)$$

Two difficulties may arise: on one hand, power networks have very large reliabilities and cascading failures are rare events; on the other hand, the computational expenses needed for the resolution of the complex models of power flows within the network become soon prohibitive, as the accuracy and level of realistic details to be included in the analysis increase.

For a computationally expensive performance function $\mathcal{G}(\boldsymbol{x})$, an accurate estimation of P_f becomes prohibitively time consuming when the failure probability is small. For example, if P_f is of the order of 10^{-p}, N_{MC} should be at least 10^{p+2} to achieve a coefficient of variation $\hat{\delta}_{MC}$ of the order of 10%.

In order to overcome this issue, many methods have been proposed in literature. In structural reliability analysis, for example, First or Second Order Reliability Methods (FORM or SORM) are commonly used [5–9]. These methods approximate the limit state function $\mathcal{G}(\boldsymbol{x})=0$ around the so-called Most Probable Failure Point (MPFP) or “design point” by a Taylor series expansion, which allows fast analytic computations of the failure probabilities. However, these methods suffer from a major drawback, i.e. they require the numerical computation of the gradient and the Hessian of the limit state function, thus potentially leading to large and not easily quantifiable estimation errors.

Other approaches increase the efficiency of the MC estimators by resorting to so-called variance reduction techniques.

Probably, the most popular variance reduction technique is that of importance sampling (IS), which has been successfully applied in many fields of research. In IS, a suitable importance density alternative to the original input pdf $f(\boldsymbol{x})$ is chosen so as to favor the MC samples to be near the failure region, thus forcing the rare failure event to occur more often. The major difficulty of the method lies in the a priori definition of a suitable importance density. In order to overcome this issue, a common approach in structural reliability is that of choosing the importance density as a joint Gaussian distribution centered around some properly identified design points [10], such as, for example, the MPFP(s) identified by a FORM (or SORM) in the isoprobabilistically transformed standard input space [9,10]: by doing so, it is possible to refine the result of the FORM (SORM) by an IS procedure, which picks the samples in the vicinity of the failure region. Another popular strategy is that of iteratively adapting the importance density by exploiting the model evaluations gathered in previous estimation steps or, in other words, to use some adaptive pre-samples [11,12]: in order to gain this prior knowledge, usually, many performance function evaluations are required to find samples falling in the failure regions, in particular when the failure probability to be estimated is very low. To overcome this problem, [13] introduced a method based on Markov Chain Monte Carlo (MCMC), based on a modified Metropolis-Hastings algorithm (or similar schemes), to adaptively approximate the optimal importance density. In general, however, the major drawback of IS-based approaches is that for complex, high dimensional problems, it is often difficult, if not impossible, to build efficient importance densities, as observed in [14] and also demonstrated in [15].

One of the most successful variance reduction alternative technique is subset simulation (SS) [16,17], which does not suffer from this issue. The method is based on a representation of the failure probability as the product of conditional probabilities of some properly chosen “intermediate”, more frequent failure events, the estimation of each of which only requires few performance function evaluations. The conditional probabilities are, then, sampled by means of a MCMC method. However, the total number of evaluations required remains too large in many applications requiring long-running computer codes [18], so that the failure probability estimation may still be computationally prohibitive. Moreover, the method's efficiency stems from i) a smart definition of the “intermediate” failure events, which is not an easy task, especially for complex models with little or no availability of prior information, and ii) the crucial choice of the proposal pdfs, which is, in general, significantly dependent on the problem under analysis, thus limiting somewhat the flexibility of the approach [16].

Another important class of variance reduction methods successfully addressing the problem of large dimensionality is that based on line sampling (LS) [19], which uses lines, instead of points, to probe the failure domain. The method stems from the determination of an important direction pointing towards the failure domain, with respect to which the sampling lines are then defined, thus giving rise to conditional, one-dimensional problems, simpler to solve. However, similarly to SS, the method still requires too many performance function evaluations in many applications. Moreover, the efficient determination of the principal direction is still an open issue, which significantly depends on the application under analysis [19].

Recently, effective strategies for further reducing the computational efforts required by small failure probability estimation have been proposed, which use a surrogate model (metamodel) for a fast approximation of the performance function within a sampling based Monte Carlo scheme. To run a metamodel is, in fact, orders of magnitude faster than the original model, thus potentially allowing significant computational savings. In this context, the Adaptive Kriging MC Sampling (AKMCS) algorithm [20] and its improved version Adaptive Kriging Importance Sampling (AKIS) [21,22] have been recently proposed, where a Kriging-based metamodel is coupled to a MC-based strategy (crude, in AKMCS, and IS, in AKIS), within an adaptive learning scheme which automatically refines the metamodel to a desired level of precision. These methods have been demonstrated efficient in estimating small failure probabilities in different engineering fields, from structural reliability [20–22] to probabilistic risk analysis of nuclear installations [23–26]. However, the most attractive feature of this class of methods, which stimulated their application in this work, is the fact that they require very little calibration when applied to different problems, demonstrating high levels of adaptability and flexibility, as opposed to most of the methods illustrated above. A further motivation for the use of Kriging in the present context lies in the successful demonstration of its applicability for approximating step-wise discontinuous functions given by [24], provided that proper care is taken in the construction of the metamodel DoE.

Thus, in the present work, we propose to exploit this feature for the analysis of the reliability of power networks subject to cascading failures, a context very different from that of structural reliability, for which these methods were first introduced. Yet, in order to be able to