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# Analyzing sensitivity measures using moment-matching technique

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ABSTRACT

Sensitivity indices are used to rank the importance of input design variables or components by estimating the degree of uncertainty of output variable influenced by the uncertainty generated from input variables or components. With the advent of highly complex engineering simulation models that describe the relationship between input variables and output response, the need for an efficient and effective sensitivity analysis is more demanding. Traditional importance measures either requires extensive random number generations or unable to measure variables interaction effects. In this article, a generalized approach that can provide efficient and accurate global sensitivity indices is developed. The approach consists of two steps; running an orthogonal array based experiment using moment-matched levels of the input variables followed by a variance contribution analysis. The benefits of the approach are demonstrated through different real life examples.

## 1. Introduction

The advancement of computing technologies and numerical approaches drove a tremendous growth of usage of sophisticated system models to assist scientific investigation. As a result, system reliability models are getting more and more complicated with large number of risk events and dependencies between these events. Moreover, engineers or scientists make use of the models to perform various tasks and decision-making by interrogating the model to predict behavior of systems under different input variable settings.

In the context of probabilistic design [1–3], one is interested in studying the effect of input uncertainty (which are characterized by statistical distribution of input variables) to the variation of output variable. The sensitivity of a model output with respect to input variable is referred to as sensitivity measure of input variable. When the sensitivity measure is used to rank model inputs based on their influence on model output, it is called importance measure [4]. The application of sensitivity analysis is found in product development where sophisticated engineering computer models are eminent [5], nuclear waste [6,7], safety [8,9] , hurricane losses [10], medical decision [11], marine ecosystem [12], and power plant maintenance decision making [13].

Once effect of input variables is determined, design improvement can be made effectively to mitigate the associated risk due to input variation. The probability distribution function assumed for each parameter then quantifies the uncertainty that is due either to lack of knowledge about the exact value of this parameter or to an actual variability of the value of the parameter [14]. The input variables could be design or process variables such as dimension, material property, or reliability of a component in a complex system. In the latter case, the focus is on system reliability variance which is widely used in probabilistic safety and risk analysis of industrial plants such as nuclear ones. For example, risk achievement worth (RAW) measure is a common importance measure [15] defined as the ratio of the conditional system unreliability if component $i$ is zero to the actual system unreliability. Other measures have also been introduced such as the ones found in [14,16,17].

In some cases, the number of variables may be too large to be managed and design improvements using all design variables are too costly. In this situation, design engineers must make decisions to act on important design variables only and neglect the less significant ones. In such cases, it is crucial to prioritize the input variables importance based on their contribution to overall output variance so that efficient design improvements can be made.

When the input-output relationship is sufficiently captured by low order polynomial models and all inputs are uniformly distributed, one can rank the importance of input variables by simply inspecting the regression coefficients. On the other hand, when the input-output relationship is highly nonlinear or the input variables follow various probability distributions, sensitivity analysis becomes a non-trivial task. Therefore, a general approach where the extension of the idea of traditional analysis of variance (ANOVA) decomposition for model interpretation is needed [18,19].

One common way used to measure sensitivity is based on measuring the impact of varying one factor on output while the other input factors are fixed at their nominal values using the finite difference in

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output provoked by individual change of input [1,20]. Such approach is referred to as one way sensitivity measures such as tornado diagram [21,22] and spider plots [22]. In both approaches, the sensitivity analysis is evaluated as the maximum deviation of output from its nominal value such as the mean or median and does not account for all possible output values that stem from output probability distribution. These measures are referred to as local sensitivity measures as opposed to global ones.

Borgonovo and Bischke [4] provided an overview of available local and global sensitivity measures. Examples of local measures are: Tornado diagrams, one way sensitivity functions, differentiation-based measures, and scenario decomposition. Similarly, research is full of different global sensitivity measures obtained through different methods such as non-parametric methods, variance-based methods, moment-independent methods, and value of information-based sensitivity methods.

Differentiation methods are based on Taylor series expansion of model output where partial derivatives of model output relative to model input is used as the sensitivity measures [7]. However, the partial derivatives can't be used alone if the input variables have different units. Borgonovo and Apostolakis [23] suggested using the product of partial derivative and difference of input variable at a fixed local point to overcome such limitation. Several researchers proposed several ways to extend basic importance measures to account for interaction by including higher order derivatives of Taylor expansion [24] Similar concept is used in reliability analysis where components importance measures are assessed based on the partial derivative of probability of system failure with respect to component probability failure provided that failures are statistically independent [25].

Local sensitivity quantification suffers from three shortcomings: first it is local since local or nominal values for input variables are used to estimate the derivative or output increase/decrease [20]. Secondly it does not capture the full interaction impact between input variables without an additional computational cost that may hinder its feasibility for models with large number of input variables such as the case in Tornado diagrams [4]. Finally it is deterministic, i.e., the occurrences of input variable values are treated uniformly ignoring shape of distribution where input variables may come from [16]. In some cases, analysts are forced to use local sensitivity measures due to lack of knowledge on model input distributions especially when dealing with new product or process.

When model input distributions are known or can be reasonably approximated, one can refer to global or probabilistic sensitivity measures where a “total effect” index that measures the total effect of input variables including all the possible synergetic terms between that variable and all other variables is considered [17,26]. Several global sensitivity measures will be presented in section two.

The paper is organized as follows. Section two discusses available techniques such as Sobol indices and correlation ratios. The widely accepted Sobol approach will be used as a benchmark performance. Section three presents our proposal while the performance of the proposed approach is studied in section four using four examples including real engineering examples such as oil sampling module and automotive engine joint sealing. Finally, conclusions are presented in section five.

## 2. Global sensitivity analysis

Global or probabilistic sensitivity analysis utilizes knowledge gained on model inputs' distributions to estimate its impact on the random model output variable distribution or its moments.

The most common approach of global sensitivity is the sample-based probabilistic sensitivity method using Monte Carlo (MC) approach. Typical steps to rank global importance of input variables for probabilistic model output using sampling-based sensitivity analysis are as follows:

1. Assign a probability density function (PDF) to each input variable.
2. Generate samples or input matrix using a certain sampling scheme and PDF's of input variables.
3. Evaluate model output to generate output distribution of response variable. When the simulation model is computationally expensive, one typically uses a meta-model to generate the outputs [27].
4. Finally, estimate the influences or relative contributions of each input variable to the output variable.

Although the previous outlined procedure is a general approach followed by almost all sensitivity measures, measures may differ in how samples are generated (step 1) or in influence estimation method (step 4).

In general global sensitivity methods can be categorized into: non-parametric methods [7,19], variance-based [18,26,28–30], moment-independent [8,17,31], and value of information-based sensitivity methods [4]. Borgonovo [1] provided a good comparison between the different uncertainty measures techniques. Non-parametric techniques measures sensitivity through input-output regression models which can be estimated using MC simulation sample of model input variables and estimation of sample output. Once input-output model is well defined, standardized regression coefficient (SCR) [7] or Pearson's product moment correlation coefficient (PEAR) can be used as non-parametric sensitivity measures. The ability of assessing influence of input variability depends on adequacy of regression model measured using model coefficient of determination $R^2$ which can be low under the presence of model nonlinearity or parameters interaction. Screening methods extends the estimation of partial derivatives around one point to several locations to identify least important model inputs while maintaining limited number of model evaluations [32,33]. Selected locations are usually determined using design of experiment (DOE) technique such as model input range segmentation [34] and sequential bifurcation introduced by Bettonvil [35] and extended by Bettonvil and Kleijnen [36].

The variance based techniques such as Iman and Hora [37] and Sobol indices [28,30] measures uncertainty based on input variables contribution to output variance and provides broader interpretation than other techniques and capable of providing insights on model structure.

Finally, moment-independent techniques provide insights on the influence that uncertain inputs have on the output distribution. Borgonovo [1] showed by example that non-informative results can be obtained when a decision maker relies on variance as the sole representative of the output uncertainty. As a result, moment-independent importance measures were proposed [8,17,31] where the entire output distribution is considered without specific reference to its moments. These moment independent measures are driven by the area between conditional and unconditional output distribution and hold independently of parameters correlation. For example, Borgonovo [1] proposed moment-independent sensitivity indicator $\delta_i$ estimated as shown below :

$$\delta_i = \frac{1}{2} E_{x_i}\left[\int |f_y(y) - f_{y|x_i}(y)| dy\right] \tag{1}$$

where $f_y(y)$is the PDF of y and $f_{y|x_i}(y)$is the conditional density function of $y$ given $x_i$ and the expected value represent the area enclosed between the conditional and unconditional model output densities obtained for a particular value of model input $x_i$. In case of model output sparsity, transformation invariance can be used to improve sensitivity measure estimation accuracy [37,38].

Since variance based techniques requires large number of model simulations, a meta-model can be used to replace the original model to make computational cost less expensive [32,33,39,40]. For example, Baltman and Sudret [41] proposed the use of sparse polynomial chaos (PC) expansions in order to build up a PC-based meta-model to be used