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Preventive maintenance optimization for a stochastically degrading system with a random initial age

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ABSTRACT

This paper investigates the optimal age replacement policy for used systems, such as second-hand products, which start their second life-cycle in a more severe environment with an initial age that is uncertain. This uncertain age is modelled as a random variable following continuous probability distributions. A mathematical model is developed to minimize the total expected cost per unit of time for these systems on an infinite time horizon. Optimality and existence conditions for a unique optimal solution are derived and used in a numerical procedure to solve the problem. Numerical experiments are provided to demonstrate the added value and the impacts of the random initial age on the optimal replacement policy.

1. Introduction

The push for sustainability and the recent economic downturn combined with the rapid development of mobile Internet transactions have fueled the growth of the market share of second-hand products. Platforms, such as EBay, Boncoin, and Amazon, facilitate the posting and the sale of products varying from simple household appliances to complex industrial machinery. Sellers and buyers are looking to make substantial gains while ensuring that the products get a second life instead of ending up in landfills. For buyers, the main concern is to get a decent performance for the price paid. For organizations involved in the business of large-scale remanufacturing and resale of second-hand products, the stakes and expectations are even higher. Therefore, they have to offer products that have been thoroughly checked, tested, upgraded and/or refurbished and are backed by attractive warranty policies [1,2].

In [3], the authors proposed a probabilistic warranty model for second hand equipment. In their paper, two warranty policies (free replacement and pro-rata) models for second-hand equipment were considered, but they ignored the maintenance cost component. To perform their analysis, the expected warranty costs are computed for monolithic components and systems of components with known and uncertain initial ages. However, [3] analyzed the warranty cost only according to the seller's point of view. To complete the latter approach, [4] considered a quality management policy for second hand equipments with extended producer responsibility. They proposed warranty models for second hand equipments from the buyers' point of view. Recently, [5] proposed an optimal warranty policy where a mixture of new and used components are used to carry out replacements upon failure for products under warranty. Their model only considered systems with fixed and known initial ages.

In many cases, the buyers are organizations looking to acquire production equipment at lower cost. After acquisition, they have to implement maintenance policies to extend the lifetimes of these second-hand systems. There exists an extensive coverage of maintenance policies for new systems in the literature [6,7]. A commonly used policy is the age replacement policy (ARP). According to the ARP, a stochastically failing unit is preventively maintained at a predetermined age or repaired/replaced at failure, whichever occurs first. The ARP policy was first proposed by [8] and has since been widely used. A large number of variations have been proposed to adapt the policy to practical considerations (See [7,9]). For the cases of unknown failure distribution, [10] and [11] proposed an empirical model of the ARP. When the initial age of the system is random, it can be estimated by Parzen's Kernel method [12]. Unfortunately the literature devoted to maintenance policies for second-hand systems is very recent and limited. [13] proposed a mathematical model to determine the optimal upgrade and preventive maintenance actions that minimize the total expected maintenance and penalty costs for used equipment under lease. The author in [14] formulated a periodic inspection/upgrade

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model under free non-renewable warranty policy with a fixed length of warranty period for a second-hand product and jointly determined the optimal number of inspections required and an optimal improvement level to minimize the expected total warranty cost from the perspective of the dealer during the warranty period. [15] proposed two periodical age reduction preventive maintenance (PM) models for a second-hand product with known age and a pre-specified length of usage. Their objective is to obtain the optimal number of times of PM action and the optimal degree of each PM action such that the total expected maintenance cost is minimized. In these models, the second hand equipment is assumed repairable and subjected to imperfect maintenance actions. The authors evaluated the expected maintenance cost per unit of time and deduced optimal policies under periodic PM and sequential PM governed by a threshold on the hazard function. More recently, the authors in [16] studied the relationship between rejuvenation/upgrade decisions of recovered end-of-life systems and the subsequent maintenance costs incurred during their second life as refreshed products. In [16], the imperfect maintenance actions are modelled according to the hybrid hazard rate model [17,18]. The system undergoes an imperfect PM action whenever its reliability reach a given threshold. A mathematical model was developed to jointly determine the optimal upgrade level and maintenance policy to minimize the long-run expected total cost. Four decision variables were considered, namely the reliability threshold, initial acquisition age, upgrade level, in addition to the number of PM intervals.

A key factor in the determination of appropriate maintenance policies for these second-hand systems is the uncertainty on their age and degradation levels. Sellers are either unable or not willing to divulge these details. Accurate evaluation of the age of the system may be impossible due to lack of failure data or maintenance records. The seller might also withhold this information in order to secure a better price. More recently, [19] and [20–22] adapted the basic ARP developed for new systems to the case of used or second-hand equipment operating in variable environments. They assumed that the initial age of the equipments was well-known and they modelled the expected cost per unit of time and derived an optimal replacement policy. However, their assumption of a known initial age is difficult to hold for second hand systems. Furthermore, by deriving an average cost per unit time over an infinite horizon, it is assumed that for each regenerative cycle the replacements are carried out by second-hand systems that have the exact same age and will operate under the exact same environment, which is rather difficult to achieve given that end-of-life/end-of-use (EOL/EOU) systems that are recovered and refurbished come with different ages and degrees of degradation. Even the best upgrade or refurbishing operations will leave them with variable and often unknown effective ages at time of sale. It is therefore more realistic to assume that the initial age of these refreshed systems is not constant and known, but random and following a probability distribution.

Given the shortcomings identified and discussed above, this paper proposes to develop a new ARP model for stochastically degrading second-hand systems with random initial age that are to be operated in a secondary environment that is more severe than operating conditions of their first lifetimes. The initial age is considered to be a random variable following a continuous distribution. Many applications can be found for this model. For example, many second-hand production equipments are used in developed countries before being transferred to developing countries for a second life under very different climatic conditions such as high humidity, high temperatures, presence of dust and corrosive atmosphere.

The remainder of this paper is organized as follows. The problem is motivated and introduced in Section 2. In Section 3, the expression of the total expected maintenance cost per unit of time is developed. This section also presents the optimization of the total expected maintenance cost in the second operating environment. In addition, the existence conditions of an optimal replacement strategy are derived. To illustrate the validity of the model and the effectiveness of the solution method, numerical experiments and the discussion of their results are provided in Section 4.

2. Motivation and problem description

In this work, we consider a system purchased from a used systems' market. According to traceability data, the system has been exploited within an operating environment (OE_1) which is assumed to be less severe than the new operating environment OE_2 where the system is to fulfil its new mission. Because of the high severity degree of the OE_2, the system failure rate grows more rapidly in OE_2 than in OE_1. Consequently, the same level of system degradation is accumulated more rapidly in OE_2 than in OE_1 given that operating A_1 time units in OE_1 is equivalent to operating for $A_2 < A_1$ time units in OE_2. A correspondence relationship between operating times in both OE is therefore needed to evaluate the equivalent age of the system when shifting from OE_1 into OE_2. To establish this correspondence relationship, let us denote by $R_i(t)$, $F_i(t)$ and $\lambda_i(t)$, respectively, the reliability function, the cumulative distribution function and the failure rate of the system with respect to OE_i (i=1,2). The correspondence relationship, hereafter denoted by $\Phi(t)$, is then derived on the basis of *statistical virtual age* concept. This concept was first defined by Finkelstein (see for example [23–25] for more details) and provides a framework to establish an age correspondence between two identical systems operated in two different operating environments by considering one of them as a baseline operating environment. According to this concept, $\Phi(A_1)$ is called the *statistical virtual age* of the system and corresponds to the age of the system immediately after starting to operate in OE_2. Assuming that the system is more reliable in OE_1 than in OE_2, i.e. $R_1(t) > R_2(t)$ for $t \geq 0$ which in turns states that lifetimes of the system in the two OE are stochastically ordered, the age of the system at the beginning of OE_2 is evaluated as a solution of the following equality [23,24]:

$$R_1(A_1) = R_2(\Phi(A_1)) \text{ and } \Phi(0) = 0. \tag{1}$$

To illustrate, let us consider Fig. 1 where the reliability functions of the system in OE_1 and OE_2 are drawn. From this figure, one may observe that $R_1(t) > R_2(t)$ which states that in OE_2, failures of the system are more likely to occur in OE_2 than in OE_1. It follows that, the degradation process is more intensive in OE_2 than in OE_1. Therefore operating A_1 units of time in OE_1 corresponds to operating for $\Phi(A_1)$ units of time in the second environment with $\Phi(A_1) < A_1$. For example, if the system is purchased with an estimated initial age $A_1 = 10$ units of time, its corresponding statistical virtual age $\Phi(A_1)$ is only of nearly about 5 units of time. This means that accumulating the same level of system degradation requires 10 units of time under OE_1 while it requires only half under OE_2. In the rest of the present paper, we will denote by A_1 the age of the system at the end of the OE_1, and by

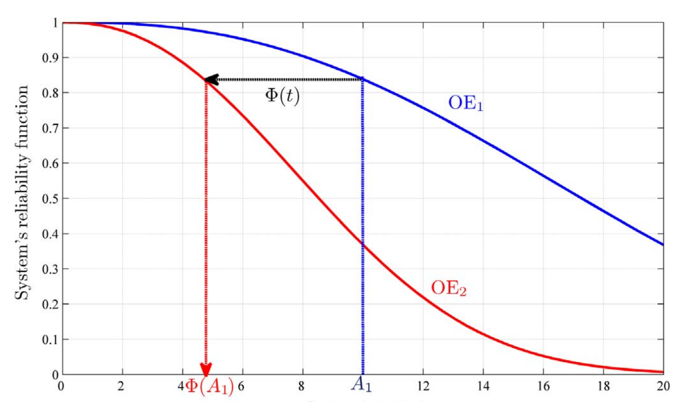

Fig. 1. Reliability function for a system under the two operating environments.