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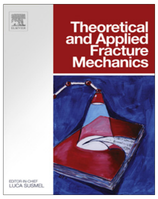

Influence of the elastic mismatch on crack propagation in a silicate-based composite

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ABSTRACT

Fracture behaviour (crack deflection) of a crack in a silicate-based composite is investigated. The three-point bending test is simulated numerically by means of the finite element method and the influence of existence of a stiff aggregate ahead of the crack tip is studied and discussed. The maximum tangential stress criterion is applied in order to calculate the initial crack propagation angle. Various geometrical configurations are investigated (the aggregate eccentricity and depth are varied) and the generalized MTS criterion based on the Williams expansion is tested considering various numbers of initial terms of the series. Unfortunately, the results show that using the generalized MTS criterion is strongly limited by the elastic mismatch caused by the presence of the stiff aggregate near the crack tip.

1. Introduction

There is always a big effort to improve the fracture mechanical properties of current construction materials. The most often used building materials are definitely silicate based (particularly cement-based) composites with their typical quasi-brittle behaviour, which has not been sufficiently explored and described yet. Moreover, their fracture mechanical properties are influenced by presence of aggregates (AGG) in the basic matrix (MTX). Thus, there exists a possibility to control/affect (in some extent) the behaviour of a crack in such material. It should be emphasised that an interfacial transition zone (ITZ) arises between the MTX and AGG and therefore, it has to be taken into account when the numerical analyses are performed. The presence of both AGG and ITZ affects the crack behaviour and generally the fracture mechanical properties of the material under study; several studies on this topic can be found for instance in [1,2] or some pilot analyses have been performed by the authors of this paper, see e.g. [3].

The above mentioned quasi-brittle behaviour of the cement-based material represents another large issue that needs to be investigated intensively. While the well-known classical linear elastic one-parameter fracture mechanics concept can be applied if the nonlinear zone around the crack tip is very small (see e.g. [4] or other fracture mechanics textbooks), the quasi-brittle material exhibits much more complicated fracture response and therefore more complex fracture theory needs to be derived. Recently, several works show that using more parameters for description of the stress field can be helpful, see e.g. [5–11]. The basic idea consists in application of the so-called Williams power series expansion [12] suggested for approximation of the stress/displacement crack-tip field. Also several authors' works, e.g. [13–16], prove that using more terms of the series expansion enables the crack-tip stress field to be expressed more effectively than if only the first term, corresponding to the stress intensity factor, is considered. The importance of the *T*-stress (second term) of the Williams expansion (WE) and of the higher-order ones is described also in papers [17–25], where the crack path is investigated.

In this paper, the initial crack propagation direction is investigated on a three-point bending cracked specimen. Based on the recent research, see e.g. [3], several parameters of the numerical model (especially the location of the AGG ahead of the crack tip) are varied in order to present a comprehensive parametric study analysing the crack behaviour. Classical linear elastic fracture mechanics approach [4,26] is applied and supplemented by its multi-parameter (generalized) form. The research introduced represents an important part of the ongoing work on testing of the multi-parameter theory that shall be suitable for more advanced fracture mechanics tasks, such as for instance describing the fatigue crack growth in presence of stress concentration, when the incremental method introduced in [27–31] can be applied.

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2. Methodology

Several basic terms and theory need to be explained before the results are presented.

2.1. Williams expansion (WE)

The fundamental theory applied in this research is the description of the crack-tip stress/displacement field by means of the Williams power series expansion [12]. The truncated form for the stress tensor/displacement vector components in a homogeneous elastic isotropic cracked body subjected to arbitrary remote loading can be written as (the polar coordinate system (r, θ) is considered to be centred at the crack tip):

$$\sigma_{ij} = \sum_{n=1}^{N} \frac{n}{2} r^{\frac{n}{2}-1} A_n f_{\sigma_{ij}}(\theta, n) + \sum_{m=1}^{M} \frac{m}{2} r^{\frac{m}{2}-1} B_m g_{\sigma_{ij}}(\theta, m) \quad \text{where } i,j \in \{x,y\}. \tag{1}$$

$$u_i = \sum_{n=1}^{N} r^{\frac{n}{2}} A_n f_{u_i}(\theta, n, E, \nu) + \sum_{m=0}^{M} r^{\frac{m}{2}} B_m g_{u_i}(\theta, m, E, \nu) \quad \text{where } i \in \{x,y\}. \tag{2}$$

The symbols in Eqs. (1) and (2) have the following meaning: E represents Young's modulus and ν represents Poisson's ratio; $f_{\sigma_{ij}}$ and $g_{\sigma_{ij}}$ are known functions corresponding to the stress distribution and to loading mode I (f) and II (g), respectively; f_{u_i} and g_{u_i} are known functions corresponding to the displacement distribution and to loading mode I (f) and II (g), respectively; N and M represent the number of the terms (of mode I and II) involved in the power series; A_n and B_m correspond to the unknown coefficients of the Williams expansion terms for mode I and mode II of loading, respectively. Because the values of A_n and B_m depend on the specimen geometry, crack length and loading/boundary conditions, they have to be calculated for each specific configuration. Note that the first terms of the Williams series corresponds to the well-known stress intensity factors and it holds: $K_I = A_1\sqrt{2\pi}$ and $K_{\mathrm{II}} = -B_1\sqrt{2\pi}$

2.2. Over-deterministic method

Based on the previous experience of the authors, the so-called over-deterministic method (ODM) [32] was chosen for estimation of the coefficients of the WE terms A_n and B_m. The ODM accuracy and reliability have been extensively tested, which is together with several recommendations on its most effective application published for instance in [32–35]. Whereas the other numerical methods (such as the hybrid crack element method and the boundary collocation method [36–39]) require the use of special hybrid crack elements or complicated FE formulations, the conventional finite element (FE) code is sufficient for application of the ODM procedure. The method is based on the least-squares formulation and requires only the knowledge of the nodal displacement distribution around the crack tip. Then, directly Eq. (2) is applied: the displacements of a selected set of nodes (often defined at a particular radial distance) around the crack tip with its polar coordinates can be substituted into Eq. (2) and the only variables, A_n and B_m can be searched. In order to fulfil the principle of the method, at least $(N + M)/2 + 1$ nodes need to be considered in order to calculate $N + M$ coefficients.

2.3. MTS criterion

In order to investigate the initial crack propagation direction, some fracture criterion needs to be applied. In this paper, the most extended maximum tangential stress (MTS) criterion is chosen [40]. The criterion is based on the stress value; particularly, it predicts that the crack will extend in the direction where the tangential stress value is maximal. This condition can be written in form of derivations:

$$\frac{\partial \sigma_{\theta\theta}}{\partial \theta} = 0, \quad \frac{\partial^2 \sigma_{\theta\theta}}{\partial \theta^2} < 0 \tag{3}$$

If the one-parameter linear elastic fracture mechanics concept is applied, the MTS criterion can be rewritten in its explicit form. The crack propagation angle can be expressed directly as a function of the stress intensity factors K_{I} and K_{II}:

$$\gamma = 2 \arctan \frac{-2K_{\mathrm{II}}}{K_{\mathrm{I}} + \sqrt{K_{\mathrm{I}}^2 + 8K_{\mathrm{II}}^2}}. \tag{4}$$

Eq. (4) is utilized very commonly in practice because of its simplicity. Nevertheless, in this work, the generalized form of the criterion is tested, i.e. the tangential stress is expressed by means of the Williams power expansion and the angle with maximal $\sigma_{\theta\theta}$ is sought.

Note that the principle of the MTS criterion was used also directly in the FE code. The values of the tangential stress were investigated at a selected distance from the crack tip and the angle with the maximal value of $\sigma_{\theta\theta}$ was determined as the direction of the further crack propagation.

3. Parameters of the numerical model

A specimen geometry for the three-point bending test was chosen for the analysis presented, see Fig. 1. The parametric study follows the previous partial analyses, see e.g. [3]. The schema of the cracked specimen with an eccentrically placed aggregate surrounded by the ITZ can be seen in Fig. 1 and the dimensions are: half specimen length $L = 80$ mm, half span between the supports $S = 60$ mm, specimen width $W = 40$ mm, crack length $a = 12$ mm, thickness of the ITZ layer $t_{ITZ} = 100$ µm, diameter of the aggregate $d_{\mathrm{AGG}} = 4$ mm. The material properties and the loading were considered: applied force $F = 1$ kN, Young's modulus of the matrix $E_{\mathrm{MTX}} = 30$ GPa (corresponds to cement paste properties), Young's modulus of the aggregate $E_{\mathrm{AGG}} = 60$ GPa (corresponds to basalt properties), Young's modulus of the ITZ layer $E_{\mathrm{ITZ}} = 55$ GPa (the value is similar to that of the aggregate, but in order to distinguish the individual materials a slightly lower value was chosen),

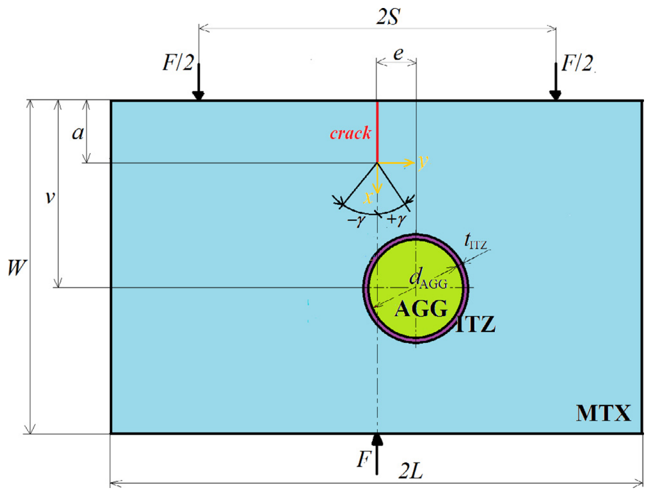

Fig. 1. Schema of the modelled 3-point-bending specimen with a crack of the length a and an aggregate with ITZ layer located ahead of the crack tip.

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