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Effects of transitional functions on multiscale fatigue crack growth

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ABSTRACT

Transitional functions are the reflection of combined effects of material, loading and geometry through the process of fatigue crack growth. They are incorporated in the segmented multiscale model. Based on the formulated multiscale fatigue crack growth model, effects of transitional functions on the crack growth behaviors of superalloys are discussed. Considering the inevitable assumption on the three transitional functions, the uncertainties of coefficient variation remains to be clarified. Material, loading and geometry effects are separately studied on the basis of collected test data of 2024-T3 Al sheet. Consequently, three transitional functions are optimally determined. These findings offer a new perspective of combined material, loading and geometry effects on crack growth behaviors of superalloys.

1. Introduction

The inherent multiscaling in materials and structures necessitates the multiscale approach in fracture mechanics [1–3]. It is universally recognized that multiscaling shall be a rule rather than an exception. Based on the theory of strain energy density [4,5], concept of scale segmentation was introduced to the multiscale crack growth model in the pioneering work of hierarchical models [2–3,6–8]. The energy-oriented model is able to overcome the monoscale limitation of stress-oriented model, which can be attributed to the variation of stress singularity orders at different scales where weak and strong singularities were located.

Considering the microscopic irregularities in materials, three fundamental variables are defined to reflect the effects of material, loading and geometry on fatigue crack growth behaviors. These variables are further modified to demonstrate the transitional effects in multiscale behaviors [9–11].

In the newly proposed multiscale model, transitional functions are further modified to be a function of fatigue cycles N. The nonhomogeneous physical property is accounted by material function. The restraining effects of loading are accounted by loading function while geometric effects are reflected through geometric transitional function. These transitional functions are assumed to be monotonically decreasing or increasing, which is formulated on the basis of physical mechanism through the process of material degradation.

It shall be pointed out that, there inevitably exist assumptions in the transitional functions. Though the general trends of these curves agree with the physical phenomena of material degradation, loading restriction and size effects, transitional coefficients in transitional functions remain to be specified. The main obstacle is determination of these scale parameters. Highlighted are the coefficients or index in the three transitional functions. Variation of these coefficients lead to the change of transitional functions. Roles of material, loading or geometry correspondingly vary through the process. The present work aims to particularly study the variation of three transitional functions and their impact on fatigue crack growth behaviors in 2024-T3 Al sheets. Followed is the discussion of the new perspective into short fatigue crack behaviors, as is inspired by the variation of transitional functions in multiscale fatigue crack growth model.

2. Fatigue crack growth model – multiscale segmentation

The multiscale fatigue crack growth model is formulated in previous work [9–12] from the theory of strain energy density. The main approach is to define the crack driving force that is the expression of incremental strain energy density factor ΔS. The expression of ΔS factor varies at different scale ranges [6]. Thus, the state of stress singularity can be explicitly illustrated. Emphasized here is the expression of ΔS at the scale range of micro-macro.

2.1. Incremental strain energy density factor

The strain energy is stored in a volume element ahead of a crack tip that is a quadratic form of stresses. It is reserved either at the

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microscopic or macroscopic scales. The flexibility and adaptability of SED makes it perfect factor for multiscaling problems. The modified form of strain energy density has been applied to materials and structures with notches and cracks [13–19]. The incremental SED factor ΔS corresponds to the incremental stress intensity factor ΔK that is taken as the crack driving force in fatigue crack growth model. ΔS is regarded as the energy released when the crack extends by the amount $r = \Delta a$ see Fig. 1. The crack continues growing when one of the following conditions is satisfied:

$$\begin{aligned} &S_1 < S_2 < S_3 < \quad \text{until instability} \\ &S_1 > S_2 > S_3 > \quad \text{until arrest} \end{aligned} \tag{1}$$

The present work is stipulated to the micro-macro scale range. Thus, the following expression of ΔS is adopted:

$$k_{ma}\Delta S_{mi}^{ma} = \sigma_a \sigma_m a_{mi-ma} \mu (1-\sigma)^2 d^{0.5} \tag{2}$$

where k_{ma} is the normalization factor that meets the requirement of dimensional compatibility on both sides. The exponent for a_{mi-ma} corresponds to $r^{-0.5}$, the order of stress singularity for the marocrack. σ_a stands for stress amplitude and σ_m is the mean stress. The three transitional functions (μ, σ, d) are incorporated in the expression of ΔS which is a combination of transitional functions as well as ordinary parameters. The transitional functions are to be elaborated later. The incremental SED factor ΔS is incorporated in the following equation:

$$\frac{da}{dN} = B(\Delta S)^m \tag{3}$$

The above crack growth rate relation is reminiscent of traditional model $da/dN - \Delta K$. Nevertheless, the difference between ΔS and ΔK is fundamental.

2.2. Transitional functions

The three transitional functions defined in the multiscale model are denoted as follows:

$$\begin{aligned} \mu &= \mu_{ma/mi} = \mu_{ma}/\mu_{mi} \\ \sigma &= \sigma_{ma/mi} = \sigma_{ma}/\sigma_{mi} \\ d &= d_{ma/mi} = d_{ma}/d_{mi} \end{aligned} \tag{4}$$

Subscript notations *ma* and *mi* represent macro- and microscales, respectively. μ_{ma} is macroscopic stiffness while μ_{mi} stands for microscopic stiffness. The ratio of macro- and micro- stiffness generates the material transitional function. it reflects the material effect through the process of fatigue degradation. σ_{ma} is the macroscopic stress and σ_{mi} refers to the local restraining stress. The ratio of macroscopic loading and local restraining stress determines the loading transitional function. d_{ma} is macroscopic length and d_{mi} is microscopic length or microscopic feature scale. Curve trends of the three functions are assumed to be monotonically decreasing or increasing.

It is generally postulated that material properties at microscopic scale are much stronger than macroscopic ones. Meanwhile the macroscopic stiffness as well as macroscopic stress degrades over the fatigue cycles. For the local restraining stress, its inherent resistance to crack propagation increases over fatigue cycles. Therefore, curves of $\mu_{ma/mi}$ and $\sigma_{ma/mi}$ are defined to be monotonically decreasing. While curve of $d_{ma/mi}$ is assumed to be monotonically increasing. This can be attributed to the irreversible process of crack propagation. Consequently, the following succinct functions over the variable of fatigue cycles N is hypothetically defined:

$$\begin{aligned} \mu &= \mu_{ma/mi} = 1 - \zeta N^2 \\ \sigma &= \sigma_{ma/mi} = 1 - \eta N^2 \\ d &= d_{ma/mi} = 1 + N^{\lambda} \end{aligned} \tag{5}$$

in which $\zeta\eta$ and λ refer to the transitional coefficients for material, loading and geometry functions, respectively. They vary over the fatigue life cycle of material degradation.

2.3. Transitional functions – variation of coefficients

It is postulated that monotonically decreasing trend in material and loading transitional functions. The curves decrease steadily at early stage while sharply at late stage. Geometric transitional function takes the opposite trend that is increasing monotonically. Each of them is a function of fatigue cycles N. The range of fatigue life is approximately evaluated from test data. Giving the size effect of specimen, the range of fatigue life shall be extended to a certain degree of 10–20%. In the present work, test data of 2024-T3 Al sheets is taken. Under the stress level of mean stress 24.5 MPa and stress amplitude 117.5 MPa, fatigue life of 2024-T3 Al sheets is assumed to be 450kc. Based on Eq. (5), there arises the following variation of coefficients.

$$\begin{aligned} \mu &= \mu_{ma/mi} = 1 - \zeta N^2, \quad \zeta = 0.45\times10^{-5}, 0.50\times10^{-5}, 0.55\times10^{-5} \\ \sigma &= \sigma_{ma/mi} = 1 - \eta N^2, \quad \eta = 0.4\times10^{-5}, 0.45\times10^{-5}, 0.50\times10^{-5} \\ d &= d_{ma/mi} = 1 + N^{\lambda}, \quad \lambda = 0.2, 0.4, 0.6 \end{aligned} \tag{6}$$

Notice material coefficient ζ vary from 0.45 to 0.55. The slight change is due to the fatigue life uncertainties evaluated from pure test data. This rule is also applied to loading coefficient η and geometry coefficient λ. Graphical description of the three transitional functions are Displayed in Figs. 2a–2c, respectively.

3. Transitional effects on fatigue behaviors of 2024-T3 Al sheets

3.1. Empirical approach – linearization of multiscale model

Test data of 2024-T3 Al sheets [20] is employed for case illustration. Displayed in Figs. 3a and 3b is the graphical description of crack length and crack growth rate versus fatigue cycles of 2024-T3 Al sheets. Details of the test can be found in Ref. [7]. The loading condition is mean stress of 24.5 MPa as well as stress amplitude of 117.6MPA. In order to achieve the empirical parameters B and m in the multiscale model, as is similar to C and n in the Paris model, linearization of the multiscale model is required. Take the log-log form of incremental strain energy density factor ΔS versus the fatigue crack growth rate da/dN, namely log*da/dN vs* log ΔS. y-intercept and slope of this line are taken to obtain the two empirical parameters B and m in ΔS model. This is the so-called approach of linearization. Take logarithmic operations on both sides of Eq. (3), it renders:

$$\log\left(\frac{da}{dN}\right) = \log B + m\log(\Delta S) \tag{7}$$

in which the exponents of B and m are two empirical parameters. log-log plots of $da/dN - \Delta S$ are displayed in Figs. 4a–4c inclusive.

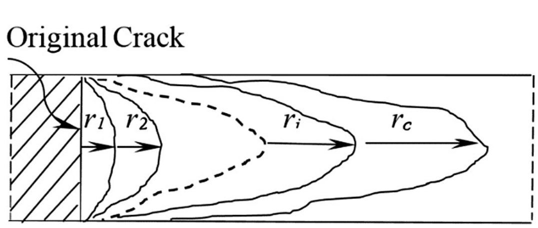

Fig. 1. Crack driving force – strain energy density factor.