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Stochastic investigation of the facture problem in functionally graded materials with uncertain mechanical properties and an arbitrarily oriented crack

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ABSTRACT

The macro and micro analysis methods of the fracture mechanics are adopted to investigate the fracture mechanics problem of functionally graded materials (FGMs) with uncertain mechanical properties and an arbitrarily oriented crack. In this paper, the piecewise-exponential model and the Mori-Tanaka model are combined to solve the mixed-mode crack problem for every sample analytically. Then according to the probability theory, the mixed-mode SIFs for the samples are obtained. So the probabilistic attributes of the mixed-mode SIFs in FGMs can be obtained. As a result, the probabilistic density function of SIFs can be provided by means of the graphic method. The method based on macro and micro mechanics can be efficiently applied to carry out the related research of the influences of the crack angles and lengths on the stochastic characteristics of the mixed-mode SIFs.

1. Introduction

Functionally graded materials (FGMs) are two- or multi-phase particulate composites in which material composition and microstructure vary spatially in the macroscopic length scale to meet a desired functional performance. This is achieved by gradually varying volume fraction of the constituent materials, which can be created by controlling over the manufacturing. This characteristic inevitably creates uncertainties in the effective material properties of a FGM [1]. The randomness in material properties and the random characteristics in fracture problems of FGMs have attracted great attention from the international research community.

During the last few decades, the mechanical properties of FGMs are usually considered as deterministic values in various theoretical and computational studies on fracture behavior of FGMs. Many published papers assumed the effective properties to be particular functions (e.g. exponential functions) [2–5]. Guo et al. [2,3] proposed the early analytical models for static and dynamic crack problems of orthotropic FGMs and FGM coating-substrate structures. Gu and Asaro [4] analyzed a semi-infinite crack in a strip of an isotropic FGM under edge loading and in-plane deformation conditions. Guo and Noda [5] gave the dynamic analytical solution of a functionally graded layered structure with a crack crossing the interface and depicted the variation of the dynamic SIFs with the nonhomogeneous parameters when the crack moves from one layer into another layer. Their work overcame basic fracture problems of FGMs with general mechanical properties. In the past decades, most of the analytical models for the crack problems of FGMs assumed the mechanical properties to be very particular functions (mainly exponential functions) so that the crack problems can be solved analytically. However, this assumption may not coincide with practice. Having considered great difference from the previous analytical models, Guo and his coauthors [6–8] proposed the piecewise-exponential model (PE model) to solve the static and dynamic crack problems of FGMs with general mechanical properties, while the PE model is based on the analytical model of a single layer with exponential properties. Hundreds of documents based on analytical models were published in the past decades on the crack problems of FGMs. The analytical models can be unified and proved to be significant with the idea of the PE model as they form a strong base for establishing a general analytical fracture model of FGMs with arbitrary mechanical properties. Guo and Noda [9] proposed an analytical thought combining with the PE model and a perturbation method to investigate the thermal shock crack problems of FGMs with general thermomechanical properties. Pan et al. [10,11] and Wang et al. [12,13] expanded the PE model to the fracture problems of FGMs with general thermomechanical or viscoelastic properties and collinear cracks.

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Chalivendra [14] developed a quasi-static mixed-mode stress fields for a crack in orthotropic inhomogeneous medium. However, there is substantial plenty of randomness in the composition of FGMs. Many investigations do not provide any measures of the stochastic behavior of FGMs [15]. To enable a better understanding and characterisation of the actual behavior of FGMs, it is obviously of prime importance that the inherent randomness in system parameters be incorporated in the analysis [16]. The phase volume fractions in practical FGMs show random fluctuations around the average trend, which leads to stochastic features of the mechanical properties of FGMs. Thus, the corresponding fracture analyses of FGMs with stochastic mechanical properties will become very complex [17]. Unfortunately, relatively little efforts have been made in the past by the researchers and investigators on research in analytical fracture problems of FGMs with stochastic properties. Shahabian and Hosseini [1] studied the dynamic response of a functionally graded cylinder subjected to mechanical shock loading with randomness in material properties by using the Monte Carlo simulation. Second-order statistics of the static response-mean and variance of transverse deflection of functionally graded plates with multiple randomness in material properties, volume fraction index and lateral loading were obtained by Yang et al. [16]. A stochastic fracture mechanics model was proposed for predicting probabilistic characteristics of SIF for a crack problem of FGMs by Guo et al. [17]. Ferrante and Graham-Brady [18] presented three multiscale models for fracture analysis of a crack in a two-phase, functionally graded composite. The statistics of temperature and thermal stress were analytically obtained in FGM plates with uncertainties in the thermal conductivity and coefficient of linear thermal expansion by Chiba and Sugano [19]. Besides, some researchers have given attention to the uncertain properties of FGMs. Lal et al. [20] provided a probabilistic tool for incorporating and handling the structural material uncertainties in the analysis of the structures. Hosseini and Shahabian [21] stochastically studied the transient thermo-elastic waves and coupled thermo-elasticity without energy dissipation based on Green–Naghdi theory in functionally graded thick hollow cylinder with Gaussian uncertainty in constitutive mechanical properties. And the stochastic hybrid method presented in Ref. [22] can be considered as an innovation for stochastic analysis with large numbers of samples. In Ref. [23], the material properties of FGMs and piezo-electric materials with volume fraction exponent were modeled as independent random variables. A numerical example involving an edge-cracked functionally graded specimen under a mixed-mode deformation was analyzed by the various multiscale and microscale models examined in Ref. [24]. In Refs. [25–27] new interaction integral methods were developed for solving the static and dynamic crack problems of nonhomogeneous materials with complex interfaces. Their contribution lies in

the domain-independence of the new interaction integral methods still stands in nonhomogeneous materials even when the integral domain intersects the interfaces under thermal loading or dynamic loading. It will provide significant support for the crack propagation simulation of FGMs or nonhomogeneous materials with complex interfaces. Then the new interaction integral methods were expanded to the nanoscale crack propagation problems [28,29]. Apart from the SIFs, the T-stress is another key parameter in fracture mechanics because it affects the crack growth direction, crack-tip constraint and fracture toughness greatly. A domain-independent interaction integral for extracting the T-stress was proposed for nonhomogeneous materials with interfaces [30]. Then it is set in the extended finite element method (XFEM) so that the T-stress can be solved with high accuracy and efficiency. If the damage and crack propagation in nonhomogeneous materials need to considered, the damage evolution method [31] can be applied.

In previous investigations, only a few of them analyzed the crack problems of FGMs considering the stochastic mechanical properties. The analytical model for these problems was developed by Guo et al. [17]. But only mode-I crack problem was considered in it. Up to now, no analytical solution of the crack problems aims to the both uncertain mechanical properties and the arbitrarily oriented crack. Usually the direction of the crack in FGMs is assumed to be particular, such as being parallel to the variation direction of the material property. In this paper, taking into account microstructural details of uncertain properties, a macro analysis method of the fracture mechanics will be applied to the fracture mechanics problem of FGMs with uncertain mechanical properties and an arbitrarily oriented crack. And the probabilistic characteristics of the mixed-mode SIFs of FGMs with stochastic mechanical properties will be studied analytically. Especially the influences of the different angles and lengths of the crack on the randomness of the mixed-mode SIFs are discussed.

2. Stochastic micromechanics description

2.1. Problem definition

Consider a two-phase FGM body with domain $D \subset \Omega^3$ and a schematic illustration of its microstructure, as shown in Fig. 1. It illustrates three disjoint FGM regions in the macroscopic length scale (x) with subdomains D_1, D_2 and D_3, where $D \in \bigcup_{i=1}^{3} D_i$. The microstructure includes two distinct material phases

phase A (black), phase B (white), either of which represents an isotropic and linear-elastic material. As shown in Fig. 1, the particle–matrix region D_1 comprises particles from phase B material (white) embedded in the matrix material which is phase A (black). The particle–matrix role reverses in region D_3, where particles and matrix are phase A (black) and phase B (white) material, respectively. In the transition region D_2, there are forming intertwined clusters of particle and matrix materials.

A representative volume element at an arbitrary point $x (x \in D \subset \Omega^3)$ characterizes material heterogeneity in the microscopic length scale [15,17]. The volume fractions of particle and matrix materials are respectively denoted by $\phi_p(x)$ and $\phi_m(x)$, each of which is constrained in the scope of 0–1 and meets the restrictive condition

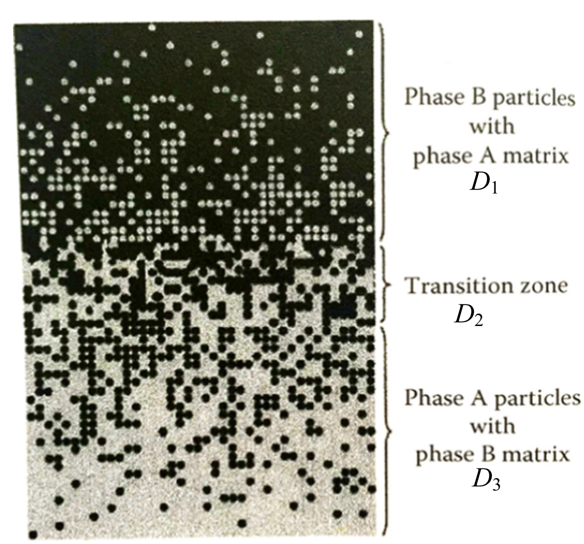

Fig. 1. Schematic of the microstructure of FGM strip.