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The Theory of Critical Distances: A link to micromechanisms

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ABSTRACT

The Theory of Critical Distances (TCD) is a methodology for prediction of the effect of stress concentration features on material failure, especially cracking-related mechanisms such as fatigue and brittle fracture. It has a long history but has become much more relevant to industrial problems in recent years thanks to the widespread availability of finite element analysis and other numerical simulation tools. An ongoing fundamental challenge is to understand the physical significance of the TCD parameters, in particular the value of the critical length L, in relation to the underlying micromechanisms of crack extension and toughening. A novel approach is presented here, in which the TCD is used to make predictions of the notch fracture behaviour of three different model materials having different crack extension mechanisms. Three different notch types (circular holes, long slots and short cracks) were investigated. In all cases the value of L was shown to be simply related to a microstructural length parameter but this relationship depended on the mechanism involved. These results provide some insights into the distinctive behaviour of different materials, especially polymers such as PMMA which display crazing behaviour, and fibre composites such as carbon/epoxy laminates.

1. Introduction

The effect of notches, and more generally stress concentration features, is potentially severe and often difficult to predict. For the extreme case of a notch which is a long, sharp crack, fracture mechanics has provided a rational approach to the problems of fatigue and brittle fracture, but there are relatively few industrial situations for which this case applies, the most obvious one being fatigue in the aircraft fuselage. More often we are dealing with cases of short cracks or notch-like features, for which fracture mechanics approaches must be significantly modified, or abandoned altogether.

The Theory of Critical Distances (TCD) is an approach which is essentially derived from linear elastic fracture mechanics, with the introduction of a new parameter, the critical distance L. L can be used in various ways to make predictions (see [1] for more details): the simplest approach, called the Point Method, which will be used throughout this article, states that failure will occur when the elastic stress at a distance $L/2$ from the notch root is equal to a critical value σ_0. When applied to predict high-cycle fatigue this remarkably simple method is capable of excellent accuracy over a wide range of materials and notch types [2]. It can also be extended to cover complex cases such as finite life, variable amplitude loading and multiaxial stress states [3]. The method has also been found to be applicable for the prediction of brittle fracture in all classes of materials [1].

Methods of this type have a long history, being first formulated for metal fatigue problems by Neuber [4] and Peterson [5], and for use with fibre composites by Whitney and Nuismer [6]. For these pioneers, a major limitation was the need for an accurate analysis of the stress field close to the notch, but this problem is increasingly being overcome by modern numerical techniques such as FEA.

Another significant barrier to the practical use of the TCD is the problem of linking the value of L to the material's failure mechanisms. It is quite easy to determine a value for L for any given material and failure mechanism simply by carrying out tests on notched specimens. In some cases L turns out to be similar to a microstructural parameter: for example we found that L is very similar to the grain size for steels failing by brittle cleavage fracture [7]. But in some cases there is no such obvious link: for example the L value for PMMA is approximately 0.1 mm even though this material has no structure larger than its molecular scale, being amorphous [1]. Likewise in composite laminate materials L values tend to be around 1 mm, significantly greater than any microstructural lengths such as fibre spacing or laminar thickness [1,6].

Further work is needed, then, to explain the values of the TCD parameters, *via* the micromechanisms which determine fracture and fatigue strength. However, it is often difficult to develop theoretical models of micromechanisms because of their inherent com-

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plexity, the number of parameters involved and the tendency, in real materials, for several mechanisms to operate simultaneously.

This paper uses a different, and simpler, approach which may be a useful starting point from which to consider this matter. Instead of using real materials, we consider hypothetical model materials on which we can carry out thought experiments. Each material has a single mechanism of crack extension by which toughness and notch strength are determined, and each has a single microstructural parameter, *d*. We can imagine inserting a notch into each material and testing it to determine the effect of the notch on measured strength. This generates a kind of experimental data which we can then compare with predictions using the TCD. In this case the Point Method was chosen as it is known to be both simple and reliable. The comparison between the resulting value of *L* and the material's *d* value may then provide insights into similar relationships to be found in real materials.

2. Model materials

Three different model materials were investigated, as follows:

2.1. Material A: Periodic barriers to crack growth

This material is illustrated in Fig. 1: it contains barriers of spacing *d* at which crack arrest tends to occur. The stress intensity required for the crack to pass through the barrier is K_{cb}, assumed to be much larger than the toughness of the rest of the material. A remote tensile stress is applied in the vertical direction, causing a pair of cracks to grow from the centrally-placed notch. This mechanism can be found in many materials where the barriers are grain boundaries, phase boundaries, osteons in bone, etc.

2.2. Material B: Cracking from brittle particles

In this material the main crack propagates *via* the initiation of a microcrack due to failure of a brittle particle, which occurs when the tensile stress on the particle reaches a critical value σ_c. The microcrack then links up with the main crack (see Fig. 2). The microstructural parameter *d* in this case is the spacing of particles. An example of this mechanism in practice is the brittle fracture of steels due to cracking of carbide particles in grain boundaries as described by Ritchie, Knott and Rice [8].

2.3. Material C: Toughening by fibre bridging

This material contains fibres of spacing *d*. Unbroken fibres pass across the crack faces near the crack tip, reducing the stress intensity (see Fig. 3). This mechanism operates in fibre composites and also in polymers such as PMMA where fibrils of molecular size form a craze [9]. It also occurs in bone where the fibres are uncracked ligaments of material [10].

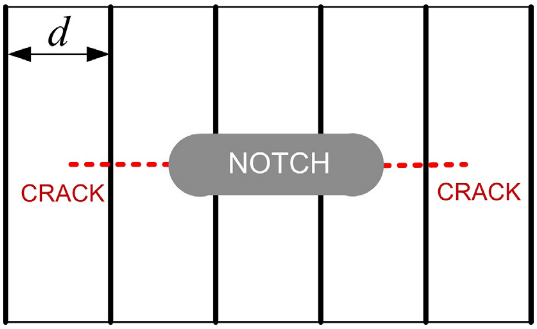

Fig. 1. Model material A.

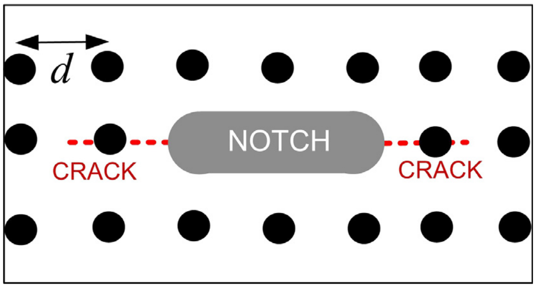

Fig. 2. Model material B.

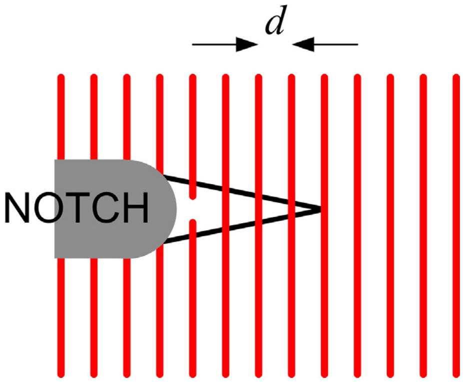

Fig. 3. Model material C (a half model is shown for clarity).

3. Notch types under consideration

Three notch types were considered:

3.1. Type (a): Circular holes

A hole in an infinite plate under tension has a stress concentrator factor $K_t = 3$. But in practice the experimentally-measured strength reduction is usually less than 3 and tends to unity as the hole size decreases [6]. Predictions can be made using the Point Method, taking the critical stress σ_o to be equal to the material's ultimate strength σ_u and using the well-known Airy stress function for the stress field around a hole. The predicted stress at failure σ_f as a function of hole radius *a* and critical distance *L* is:

$$\sigma_f = \frac{\sigma_u}{\left(1+\frac{1}{2}\left(\frac{a}{a+L/2}\right)^2+\frac{3}{2}\left(\frac{a}{a+L/2}\right)^4\right)} \tag{1}$$

3.2. Type (b): Long narrow slots

Consider a slot similar to that shown in Fig. 1, but having a length $2a$ much greater than its root radius ρ. If such a slot is used to measure the material's fracture toughness K_c, assuming the notch to be a crack of the same length, then the measured K_c, denoted K_{cm} will be a function of ρ, tending to increase as ρ increases. Point Method predictions can be made using the stress field derived by Creager and Paris [13], the result is:

$$K_{cm} = K_c \frac{(1+\rho/L)^{3/2}}{(1+2\rho/L)} \tag{2}$$

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