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A singular planar element with rotational degree of freedom for fracture analysis

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ABSTRACT

A triangle singular planar element with corner node rotational degree of freedom (DOF) is developed for crack tip field analysis. On this element's two sides adjacent to crack tip, quarter point nodes with only tangent DOF are employed to generate tangential $r^{1/2}$ displacement. Meanwhile, interpolated by both corner nodal translational and rotational DOFs, a normal displacement with $r^{1/2}$ term is constructed on these two sides. On the non-singular side of this element a displacement function equivalent to that of Allman element is also proposed. Singularities predicted by this singular element have been proven obviously more accurate than those predicted by 5-node quarter-point element (QPE). For planar problem, though accuracy of this singular element is slightly inferior to that of 6-node QPE under coarse mesh, it rapidly approaches the latter with mesh refining. When applied in shell structure, models employing the proposed element can achieve more accurate results than those using 6-node QPE even with less total DOFs.

1. Introduction

The DOFs of an ordinary flat plate element include in-plane displacement and deflection which are considered independent to each other under small deformation condition. Thus, each node of ordinary flat plate element has five DOFs, i.e. two translational displacements responding to the in-plane load; deflection and its two slopes responding to bending and twisting loads. Its in-plane deformation is actually just same as a planar element. Flat plate elements will not be coplanar when they are used in spatial curved or folded structure. In this scenario, six DOFs are required at each node in global coordinate system. The mismatch of DOF number between global and element local coordinate systems might lead to singularity of the assembled equilibrium equations and decrease of precision [1,2]. Therefore, it is considerable useful to introduce in-plane rotational DOF into the planar term of flat plate element when it is used in the analysis of folded, branched, or other non-coplanar shell structures. Even for a planar element, adding rotational DOF can obviously improve its accuracy. Some significant results have been achieved in this subject area since 1980s. Allman [3] and Bergan and Felippa [4] independently proposed a compatible triangular element with rotational DOFs; then Cook [5] extended this triangular element to a quadrilateral element. The original Allman element had spurious zero energy mode, but this previous drawback was overcome by his later researches [6,7]. Based on Allman's conception, a lot of papers have been published on rotational DOF planar elements. Wisniewski and Turska [8] enhanced Allman functions for finite deformation. Meanwhile, in-plane shear lock phenomenon was prevented in their new model. Zhang and Kuang [9] developed an 8-node isoparametric element with nodal rotations. In their element the normal displacement is a superposition of Allman's quadratic function and a new cubic function interpolated by mid-node rotation. Huang et al. [10] introduced a displacement constraint into triangular Allman element to define a rotation in term of elasticity theory and to eliminate the spurious zero energy mode. In order to improve accuracy, some approaches employed cubic function to describe displacements caused by nodal rotations. For example, in literatures [11,12] two kinds of 20-DOF quadrilateral elements were suggested; a later work [13] developed a 12-DOF quadrilateral element. These three elements are all conformed from 12-node cubic Serendipity element and have improved accuracy compared with early works [14,15]. In addition, hybrid methods [16–20] or generalized conforming techniques [21,22] were employed to construct planar element with rotational DOF.

Finite element method (FEM) has been widely applied in fracture analysis. Conventional elements employ bilinear or quadratic interpolated displacement functions. Thus, high density mesh is required near crack tip in order to represent the high displacement gradient there. To avoid dense mesh discretization around crack

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tip, Henshell and Shaw [23] and Barsoum [24,25] independently developed the quarter-point element (QPE) by moving two mid-side nodes of 8-node isoparametric element to the quarter points of element sides. The QPE can directly represent the $r^{-1/2}$ singularity of strain (stress) field near crack tip with coarse mesh. Literatures [26,27] studied the influences of displacement extrapolation procedures and mesh configurations on the accuracy of the stress intensity factor (SIF) predicted by QPE. The original QPE has been extended and refined in several ways [28,29]. Particularly, the singular element is also developed through smoothed finite element method (S-FEM), the purpose of which is to convert the area integration into line integration to avoid the derivative of shape functions. Liu's group has established a series of 5-node singular triangle elements (T5) based on S-FEM [30–32]. The 3-node triangular element linked with these T5 elements can be generated automatically to adapt crack propagation. Owing to the algorithm of S-FEM, T5 elements possess excellent accuracy and have been proven adaptive for various fracture problems, such as anisotropic material fracture [33–35], plastic fracture [36], dynamic fracture [37], and crack propagation [38,39]. Similarly, in the frame of conventional FEM, the 5-node [40] and 10-node [41] QPEs which can be compatible to linear elements were also developed for planar and spatial problems respectively.

Currently, the singularities of all QPEs are interpolated by translational DOFs. If element with in-plane rotational DOF is used in fracture analysis, introducing in-plane rotational DOF into the conventional singular element is a consequent consideration. In this paper, some essential issues for combination of Allman conception and QPE method are discussed firstly. Next, the theory and algorithm of a new triangle singular element with rotational DOF is proposed. Note that, the proposed element is developed in the frame of conventional FEM, so available abundant of FEM algorithms (or elements) can be combined (or linked) with it conveniently. Finally, accuracy of the proposed element is compared with those of 6-node and 5-node [40] QPEs through some typical fracture models.

2. Construction of $r^{1/2}$ displacement on side of triangle element with rotational DOFs

Moving mid-side nodes of isoparametric element can construct stress (strain) singularity. For example, two-dimensional QPE with $r^{-1/2}$ singularity is derived from standard 8-node isoparametric element. As shown in Fig. 1, nodes 1, 2, and 3 which originally locate on one element side equidistantly have normalized coordinates of $\xi = -1$, 0, and 1 respectively. Node 1 is postulated to coincide with crack tip. For an arbitrary point on this side, its distance r to crack tip and its displacement u can be interpolated from nodal locations (r_1, r_2, r_3) and displacements (u_1, u_2, u_3) of these three nodes:

$$r = \frac{1}{2}(\xi^2 - \xi)r_1 + \frac{1}{2}(\xi^2 + \xi)r_2 + (1 - \xi^2)r_3 \quad (1)$$

$$u = \frac{1}{2}(\xi^2 - \xi)u_1 + \frac{1}{2}(\xi^2 + \xi)u_2 + (1 - \xi^2)u_3 \quad (2)$$

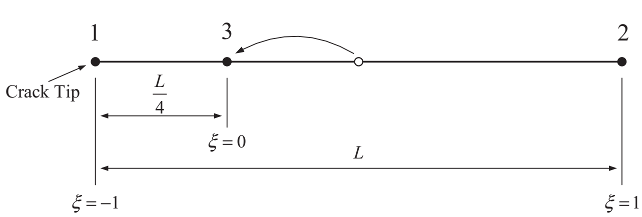

Fig. 1. Nodes on crack tip adjacent side of quarter-point element.

The subscript in Eqs. (1) and (2) denotes the node number. By distorting the mid-side node to the quarter position, Eq. (1) becomes:

$$r = \frac{1}{2}(\xi^2 + \xi)L + (1 - \xi^2)\frac{L}{4} \quad (3)$$

Here, L is the length of element side. From Eq. (3), normalized coordinate of this point can also be expressed by its distance to crack tip:

$$\xi = 2\sqrt{\frac{r}{L}} - 1 \quad (4)$$

Substituting Eq. (4) into Eq. (2), the following expression of displacement can be achieved:

$$\begin{aligned} u &= u_1 + L^{-1/2}(-3u_1 - u_2 + 4u_3)r^{1/2} + L(2u_1 + 2u_2 - 4u_3)r \\ &= a_0 + a_1 r^{1/2} + a_2 r \end{aligned} \quad (5)$$

The three terms in right side of Eq. (5) represent the rigid body motion, $r^{-1/2}$ type singularity of strain, and constant strain mode, respectively. The widely used QPE is a triangle element with six nodes, and this element can be obtained by collapsing one side of quadrilateral QPE into the crack tip point. The 6-node QPEs are usually arranged around crack tip and 8-node isoparametric elements are used outside QPEs, as shown in Fig. 2(a). In order to be compatible to 4-node isoparametric element outside, the mid-side node of 6-node QPE can be removed and the element will degenerate to a 5-node QPE [40], as in Fig. 2(b). In 5-node QPE, the order of displacement interpolation along the element side without mid-side node degenerates from quadratic to linear. So it achieves higher computational efficiency but with cost of some accuracy. Compared to the low order ordinary element (e.g. 4-node isoparametric element) and the high order ordinary one (e.g. 8-node isoparametric element), Allman type elements have an intermediate accuracy, or even reach the same accuracy level of high order element in some cases [3,5]. Meanwhile, because the Allman element has no mid-side nodes, the computing cost of a model using it will be much less than that using 8-node isoparametric element.

Whether in Allman's original triangle element [3] or Cook's quadrilateral element [5], the assumed displacement functions at element side are identical and their explicit expressions in normalized coordinate are:

$$\begin{aligned} u_n &= u_n^0 + u_n^{\theta_i} + u_n^{\theta_j} \\ &= \frac{1}{2}(1 - \xi)u_{ni} + \frac{1}{2}(1 + \xi)u_{nj} + \frac{1}{8}(1 - \xi^2)\theta_i l_{ij} - \frac{1}{8}(1 - \xi^2)\theta_j l_{ij} \end{aligned} \quad (6)$$

$$u_t = \frac{1}{2}(1 - \xi)u_{ti} + \frac{1}{2}(1 + \xi)u_{tj} \quad (7)$$

Here, the subscripts of n and t denote normal and tangent directions, respectively; subscripts of i and j denote the numbering of corner nodes; l_{ij} is the length of element ij-side (element side connecting node i and node j). Normal displacement u_n is a superimposition of three functions (u_n^0, $u_n^{\theta_i}$, and $u_n^{\theta_j}$). u_n^0 is interpolated by corner nodal translational displacements u_{ni} and u_{nj}; the two latter ones are contributed from corner nodal rotations θ_i and θ_j respectively. The tangential displacement u_t which is independent to corner node rotations is assumed merely determined by nodal tangential displacements of u_{ti} and u_{tj}. The sum of $u_n^{\theta_i}$ and $u_n^{\theta_j}$ in Eq. (6) is defined as u_n^{θ} and it can be analogical to the displacement of 8-node isoparametric element which is caused by mid-side nodal normal displacement (equivalent to $(l_{ij}\theta_i - l_{ij}\theta_j)/8$ quantitatively) on ij-side.

Under small deformation condition, relations between rotations and normal displacement slopes can be expressed as $(\partial u_n/\partial s_{ij})_i = \theta_i$ and $(\partial u_n/\partial s_{ij})_j = \theta_j$ at nodes i and j on ij-side of element, respectively. Here, s_{ij} is the local coordinate of element side. Together with the