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Predicting fatigue crack growth evolution via perturbation series expansion method based on the generalized multinomial theorem

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ABSTRACT

In this study, a novel numerical calculation method is proposed to investigate the fatigue crack growth evolution in aluminum alloy sheets accounting for the measurement error. Unlike the deterministic numerical method, the initial crack length is considered to be a modified parameter with a small correction term due to the measurement error; the solution to the crack growth equation is expressed in the form of a perturbation series after introducing said small parameter. By combining the proposed perturbation series expansion method (PSEM) with the deterministic crack growth equation, a series of modified equations for predicting the crack length history are derived based on the generalized multinomial theorem. Further, by substituting the initial condition under perturbation into the modified equations, variations in crack length versus the cycle number can be obtained. The proposed method is verified by comparing numerical results with experimental data, and the results demonstrate that the proposed model is indeed feasible and effective for predicting fatigue crack growth evolution.

1. Introduction

The fatigue damage effect on most engineering structures under repeated or cyclic external loads is one of the most significant mechanisms of deterioration [1,2]. Defects are formed inevitably during the manufacturing and fabrication stages which increase stress to initiate cracks. Considering the various service environments of the components, the impact of fatigue damage on the whole structure can vary from minor degradation to complete failure [3]. The crack length history, as shown in Fig. 1, can be measured from experiment or through mathematical integration of the two-parameter equation [4] and plays an important role in allowing time for repair or replacement of the component as necessary [5]. Thus, in recent years, many researchers have attempted to establish crack growth equations to predict the crack growth history of structures under repeated or cyclic loading.

Generally, the fatigue crack growth rate is associated with the stress intensity factor according to the principle of linear elastic fracture mechanics. Paris et al. [6] associated crack growth rate $\mathrm{d}a/\mathrm{d}N$ to the maximum stress intensity factor $K_{\max}$ in a paper published in 1961. Based on research by Liu [7], the crack growth rate was related to the stress intensity factor range ΔK. The similar relationship was established by Paris and Erdogan [8], which led to the well-known Paris equation. In recent years, a variety of modified expressions of the classic Paris equation have been presented accounting for different conditions [9], including stress ratio and the maximum stress intensity factor effect [10,11] and crack closure [12,13].

Despite these valuable contributions to the literature, a reliable prediction or simulation method for crack growth under complicated service loadings remains elusive [14]. Based on the previous work [5], measurement errors and uncertainties in the model and data are the roots of this issue. For instance, to predict crack growth evolution from the crack growth equation, one needs to solve the differential equation with a known initial crack length; thus, the initial crack length is highly significant in terms of appropriate numerical calculation of crack growth life. The initial crack length of engineering structures can be obtained via nondestructive inspection, but doing so neglects measurement errors in the initial crack length due to detection level limitations. That is to say, the initial crack length measurement data will always contain uncertainties due to measurement error [15]. Thus, assuming the deterministic value of the initial crack length is a weak point of this approach to fatigue damage tolerance; a more reliable numerical calculation procedure is yet needed [16].

To date, several researchers have conducted theoretical studies on the measurement error in initial crack length; the initial crack length is usually treated as a basic random variable subjected to probabilistic methods. Proven [16] found that the two-parameter

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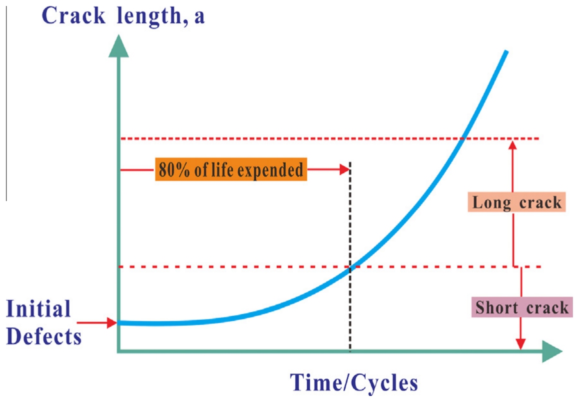

Fig. 1. Schematic illustration of crack length versus time/cycles.

Weibull distribution best describes the distribution of the initial crack length, while Besuner and Tetelman [17] assumed the distribution of the initial crack length is approximately lognormal with the standard deviation. Zheng and Ellingwood [18] found that the initial crack length is normally distributed due to measurement noise with respect to the true size of a crack as-detected. Zhang and Mahadevan [19] presented a Bayesian procedure to quantify the uncertainties in distribution parameters including the initial crack length. Ray et al. [20] extended the lognormal-distributed crack length model of fatigue crack propagation based on asymptotic analysis of growing fatigue near-crack-tip fields in damaged materials. However, this work was focused on the inherent material uncertainties while unknown initial conditions (including the initial crack length) related to the measurement instrument's precision were not considered. An important problem inherent to the probabilistic approach is that the probability distribution function must be constructed with a large amount of statistical information [21–23], while sufficient information regarding uncertainties is often difficult to obtain [24–26].

Comparatively, the perturbation method [27,28] is relatively simple and efficient. It uses an artificial small parameter for the numerical analysis of complex systems without necessitating uncertain information. It has been applied to structural response analysis [29], structural eigenvalue problems [30], and model updating [31], but has been only rarely applied to predicting crack growth evolution. Stepanova and Igonin [32,33] applied the perturbation technique to analyze the growth of fatigue near-crack-tip fields in a damaged material, but mainly focused on the asymptotic expansions of the stress and strain tensor components while neglecting perturbation in the crack length caused by measurement error.

The main contribution of this paper is the perturbation series expansion method, which is proposed to quantify the initial crack length with consideration of measurement error (not dependent on the probability distribution function) and to reliably predict crack growth evolution. Perturbation in the initial crack length is considered by introducing an artificial small parameter and the asymptotic expansion of the crack length is determined accordingly. A series of modified crack growth equations can be derived based on the generalized multinomial theorem, which permits the engineer to obtain the crack length history (crack length versus loading cycles).

The remainder of this paper is structured as follows. The basic crack growth approach is described in Section 2, and in Section 3, the corrected crack growth equations are obtained based on the perturbation series expansion method (PSEM). Three examples are accomplished to demonstrate the feasibility and effectiveness of the proposed approach in Section 4, and Section 5 provides a brief summary and conclusion.

2. Basic crack growth rate equations

The linear elastic fracture mechanics (LEFM) approach has been established as an effective theoretical tool to investigate crack growth processes [34]. In 1958, the stress-intensity factor K was first introduced by Irwin [35] for static fracture analysis. See the following equation:

$$K = \beta(a) \cdot \sigma \cdot \sqrt{\pi a} \tag{1}$$

where a is the crack length, σ is the far-field stress, and $\beta(a)$ is the geometric function with respect to the possible stress concentration; the stress-intensity factor K depends on all these parameters simultaneously. Based on Irwin's concept, Paris and Erdogan [8] established a power-law relationship as:

$$\frac{da}{dN} = C \cdot (\Delta K)^n \tag{2}$$

where da/dN is the fatigue crack growth rate, ΔK is the stress intensity factor range, and C and n are material-specific constants that are determined experimentally and listed in Table 1 as a few example values for typical materials.

A schematic representation of da/dN versus ΔK for the typical crack growth behavior in metals is illustrated in Fig. 2, where the relationship between crack growth and ΔK can be divided into three regions [19]. The crack growth is slow in region I while region III is related to the rapid crack propagation; in region II, known as the "intermediate crack growth" or "Paris region", the Paris law and its variants can be applied for the prediction of the crack growth history.

The stress intensity factor range ΔK in Eq. (2) can be obtained as:

$$\Delta K = K_{max} - K_{min} \tag{3}$$

where K_{max} and K_{min} are the maximum and minimum stress intensity factor. Based on LEFM theory, ΔK can be derived as follows:

$$\Delta K = K_{max} - K_{min} = \beta(a) \cdot (\sigma_{max} - \sigma_{min}) \cdot \sqrt{\pi a} \tag{4}$$

Table 1
Crack growth data for various technical materials [34].

$da/dN = C \cdot (\Delta K)^n$	C	n
Steel	5.79×10^{-11}	2.25
Aluminum alloy	9.82×10^{-12}	3
Titanium alloy	3.56×10^{-15}	4

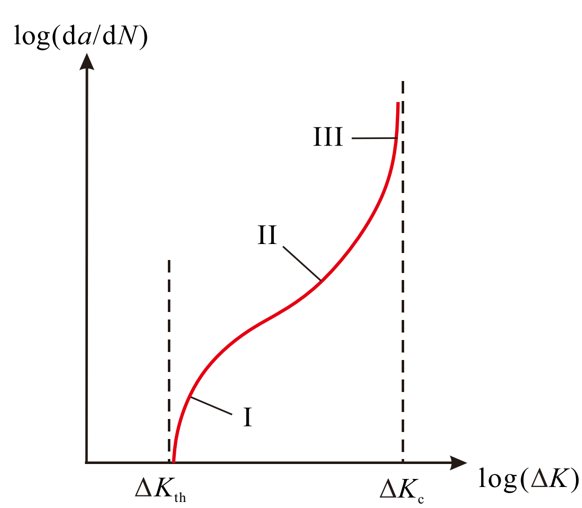

Fig. 2. Schematic diagram of the relationship between crack growth rate and ΔK.

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