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Solution of two-parameter cohesive law using Chebyshev polynomials for singular integral equation

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ABSTRACT

Complex stress and displacement potentials are used to examine the effect of cohesive stress on crack opening and stress around a two-dimensional slit-like crack in an isotropic material. A two-parameter right-angled triangular relationship between the cohesive stress and crack opening is assumed to hold. Following an earlier work, the problem is reduced to a singular integral equation (SIE), which is solved numerically using Chebyshev polynomials. The results quantify the effect that the cohesive stress has on the crack opening, and on the stress field around the crack. Finally, it is also shown that the region of action of cohesive stress ("cohesive zone") comes out as a part of the solution based on the choice of the two parameters. The presented method is more direct than the commonly used variational methods.

1. Introduction

Theories for predicting crack propagation in brittle elastic materials have been advanced by Griffith [1], and augmented by Irwin [2] and Barenblatt [3]. These works are considered fundamental in the field of fracture mechanics. The energy-balance criterion of crack growth [1] has been generally accepted by fracture mechanicians to be the most appropriate test to determine the crack initiation in a body that is loaded by external forces. However, this theory ignores the supposedly unrealistic prediction of infinite stresses at the crack tip. Though linear elastic fracture mechanics has been shown to be valid very close to the crack tip (1 nm for a silicon crystal) for brittle materials [4], the macroscopic continuum level treatments in fracture mechanics are riddled with infinitely large stresses near the crack tip [5]. Barenblatt [3] attempted to resolve this issue by considering the so-called cohesive stress (between the atoms/molecules) on opposite crack faces, arguing that it will cause the stress at the crack tip to be finite. In its original conception, the cohesive force was assumed to be an attractive force that acts between the two faces of a crack that are in close proximity, but not in actual contact.

This work was immediately picked up by many Soviet researchers of the time. The ideas contained in Barenblatt's work had been extended by researchers to many domains of fracture mechanics, such as viscoelastic media [6] and thermal stresses [7]. Applications of the cohesive stress theory also appeared in hydraulic fracturing [8] and time-based fracture [9]. Attempts had been made to study the thermodynamics of cohesive stress theory for arriving at the condition of crack initiation [10]. Extensions to Barenblatt's theory have been proposed for a deeper understanding [11,12].

As time progressed, the availability of powerful computers allowed the use of numerical techniques in the application of the theory of cohesive stress in a crack [13–15]. Cohesive stress theory forms a part of commercially available softwares these days, proving that it has become a mainstream serving industry and academics alike. Through its huge popularity and usage, it continues to draw applications in self-healing materials [16], hydrogen embrittlement [17], inertia of a propagating crack [18], multiscale simulation of dynamic fracture [19], etc.

It is not difficult to observe that in almost all earlier works on the cohesive stress, the prime question is related to crack growth initiation and progression. As the cohesive stress acts by virtue of the separation between the crack faces, it always acts – not only when the crack grows or is about to grow. Despite the long history of interest in the concept of cohesive forces, there have been very few analytical investigations of the effect of this force without its role in crack initiation or growth, for example Parihar [20] had done investigations about crack opening by spatially varying internal pressure. The present work deals with cohesive stress which vary with the crack face separation (opening). In the present work, the crack is assumed to neither grow nor be at a stage at which it is

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just about to grow, so that the focus remains on the cohesive stress and its influence on the crack opening and the stresses around the crack. A brittle, linear elastic isotropic material is considered. The infinitely large body is given a remote biaxial loading, and a thin slit crack is considered with *almost* zero initial crack opening. It is possible to derive an expression following an earlier work [21], in the form of a singular integral equation (SIE), for the crack opening subject to a general cohesive law (ie. a cohesive traction-separation relationship). However, the solution of this equation will require selection of a specific cohesive law. SIEs have been solved in fracture mechanics [22–27] but only constant stress on the crack faces has been considered. In the present situation, as will be seen, the two-parameter cohesive law is such that the cohesive stress linearly varies with the crack opening bound by a cut-off maximum value. It will be shown that Chebyshev polynomials are capable of solving such cohesive laws in SIE and giving insights into the stress field in the crack plane.

The exact nature of the cohesive law is an important parameter in fracture mechanics for general boundary conditions and loading [28]. Moreover, a complete understanding of the impact of the cohesive stress on the crack plane stress requires the need for a well-defined cohesive law. A clever way to get out of this fundamental problem is to assume that the cohesive force is related to the internal or surface energy [29]. Some researchers assume a simple law and then attempt to prove that it is reasonable [30]. Others appeal to micromechanics [31], atomic/molecular [32] and experiments [33] to derive the cohesive law. The application of these cohesive laws range from loading-unloading hysteresis [34] to carbon nano-tubes structures [35]. In the present work, a two-parameter right-angled triangular cohesive law is assumed to act between the crack faces.

A review of the literature in this field proves that the need and reason for consideration of the cohesive stress in a crack has been debated, and different accounts have been given. For example, it has been argued that the cohesive stress theory and the Griffith theory lead to identical predictions of the equilibrium crack length for small cohesive zones [5]. Others insist that Griffith was not missing anything from his theory, but had implicitly included cohesive stress in a different form [36]. The concept of the cohesive stress has been used as a simplification in the process zone [37], although it must be noted that a cohesive force (as a distance-dependent force) exists even in the absence of the process zone or plasticity. A more convincing motivation is that the singularity in the crack opening stress is remedied by assuming the existence of the cohesive stress near tip, where an attractive force between the faces of the crack balances the elastic stress [38]. This is confirmed in the present work.

The now classical, and most widely used, Finite Element Methods (FEM) to solve cohesive crack problems are based on the variational principle where the original problem is converted to a variational form which is then solved numerically. The presented method to find the crack profile and stress in the crack plane is arguably advantageous as it is more direct (lesser approximations involved). In FEM techniques, the minimization of energy (sum of strain energy and fracture energy) functional is done. The existence of solutions of such problems has been mathematically questioned [39]. This may also motivate the use of non-variational based methods to study this problem – the presented SIE method being one of them. It will be shown that this method offers rigorous derivation and highly accurate results very close to the crack tip.

The region in the crack plane where the cohesive stress acts ("cohesive zone") has no clear boundaries. Barenblatt [3] hypothesized that the cohesive force acts between the crack faces for a short region near the crack tip. However, some researchers choose to ignore the cohesive stress acting between the crack faces, and consider it only in the small process zone ahead of the crack tip [40], whereas others take it to extend throughout the crack plane [41]. A more convincing region of action of cohesive stress was brought forward [42], which distinctly demarcated the region of action of cohesive stress and the yield stress in a ductile material. In the present work, it will be shown that the cohesive zone is implicitly determined when the two parameters in the cohesive law is chosen. Meaning, the traction-separation cohesive law decides for itself the region where the cohesive stress will be acting. The cohesive zone is shown to contract to the crack tip for certain range of parameters – the physical significance on the stress field is discussed for the same case.

The nature and region of action of the cohesive stress in a general crack is defined in the next section, before specializing to a thin slit crack to derive a singular integral equation. This equation will be solved using Chebyshev polynomials, which will be validated before deriving and studying the results.

2. Nature and region of action of cohesive stress in a crack

The cohesive force is the attractive force between the atoms on the opposite faces of the crack. The nature of this force maybe ionic, or van der Waals, or any other special kind, but it will be called *cohesive* as long as it is attractive in nature and resists the increase of crack opening with the application of remote tensile load. All the attractive forces in the crack plane are shown in Fig. 1. The dashed (- -) forces are already accounted for in linear elasticity, therefore only the solid (–) ones are referred to as cohesive forces. The cohesive force will depend on the crack opening and hence may act strongly or not at all at certain points depending on the crack opening. These cohesive forces are not included in classical linear elastic fracture mechanics. The attractive forces in the region ahead of the crack tip are therefore already included in the linear theory of elasticity, and need not be separately considered. This is universally true for a crack in equilibrium that does not intend to grow on the application of a small remote loading.

For a crack that does intend to grow, the definitions of the surface energy and surface tension [43] will be useful for a clearer understanding of the nature of cohesive stresses in a crack. The surface energy γ of a solid is defined as the work γdA needed to reversibly and isothermally create an elemental area dA of the new surface. This may be done by overcoming the weak van der Waals forces or the strong ionic forces, both of which depend on the distance between surface atoms on opposite faces of the crack. Meanwhile, the asymmetry of interactions at the surface of a solid causes a modification in the ordered arrangement, reflected in

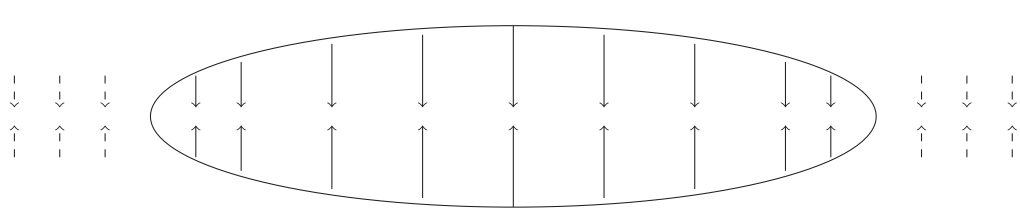

Fig. 1. Attractive forces along the crack plane.