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Stress intensity factors for mixed mode fracture induced by inclined cracks in pipes under axial tension and bending

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ABSTRACT

Mixed mode fractures can be triggered by either multi-axial stresses or inclined cracks or both. Little research has been undertaken on stress intensity factors for inclined cracks in pipes under axial tension and bending. This paper intends to determine stress intensity factors for mixed mode fracture induced by inclined external surface cracks in pipes under axial tension and bending, using a combined *J* integral and 3 dimensional finite element method. Formulae for the influence coefficients of stress intensity factors are developed. To predict the fracture failure of the pipe, the propagation of inclined cracks in pipes is also investigated. From numerical results, it is found in the paper that, for given wall thickness to internal radius ratio and crack depth to half crack length ratio, the absolute values of the influence coefficients of all three modes I, II and III stress intensity factors along the whole crack front increase with the increase of the relative depth for all inclination angles of the surface cracks. It is also found that the crack propagation angles in depth of the inclined cracks increases in magnitude along the whole crack front with the decrease of the crack inclination angle defined as the angle between the crack and pipe axial direction. The results presented in the paper can equip practitioners for more accurate prediction of pipe failures under various loads.

1. Introduction

Surface cracks have long been recognized as a major cause of potential failures for cylindrical structures, e.g., pipes. This is because, under external loads and exacerbated with material deterioration, these cracks can quickly grow and the stresses around the crack region intensify, which may result in sudden failure of the structure. To prevent catastrophic failures of cracked pipes, it is essential to accurately determine the stress intensity factors for these cracks. A literature review on stress intensity factors for surface cracks in pipes suggests that most current studies focused on single Mode I fracture, where Modes II and III stress intensity factors are assumed zero along the crack front. In engineering practice, however, surface cracks in pipes or hollow cylinders may appear in longitudinal, circumferential or inclined directions. They are also, in most cases, subjected to multi-axial stresses, e.g., normal, in-plane shear and out of plane shear stresses. As such the mode of fracture of cracked pipes is more likely to be mixed rather than single mode. As it is known, there are three modes of fracture, namely Modes I, II and III [1]. In Mode I, the applied load is normal to the crack plane and tends to open the crack. Mode II refers to in-plane shear mode and tends to slide one crack surface with respect to the other, while Mode III corresponds to out-of-plane shear. Mixed mode could be a combination of any two or three of the above three modes.

Mixed mode fractures occur mainly under two conditions: (1) stress state on the crack surface is bi/multi-axial; (2) cracks are inclined (even under uniaxial stress state). For pipelines, the longitudinal or circumferential surface cracks subjected to two or three of the opening, in-plane shear or out-of-plane modes meet the first condition for mixed mode fracture. Ayatollahi and Khoramishad [2] studied the effect of soil weight on the stress intensity factors for a longitudinal semi-elliptical crack imbedded at different circumferential positions in a buried pipe under bi-axial stresses. Shahani and Habibi [3] investigated the problem of mixed mode fracture induced by a semi-elliptical circumferential external crack in a hollow cylinder under axial force, bending moment and torsion. Predan et al. [4] presented numerical studies of Modes II and III stress intensity factors of a semi-elliptical surface crack in a hollow cylinder subjected to pure torsion.

The mixed mode fracture induced by the orientation of surface cracks in pipes, i.e., inclined cracks, satisfies the second condition for mixed mode fracture. Inclined cracks can be caused by

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Nomenclature

a	depth of a semi-elliptical surface crack
b	semi-major axis
B_{11}, B_{22}, B_{33}	diagonal elements of coefficient matrix
c	half-length of surface semi-elliptical crack
d	wall thickness of a pipe
E	Young's modulus
E'	$=\frac{E}{1-\nu^2}$ for plane strain, $=E$ for plane stress
$\boldsymbol{F}_A, \boldsymbol{F}_{IP}, \boldsymbol{F}_{re}$	influence coefficients for axial stress, internal pressure, and a combination of both respectively
F_I, F_{II}, F_{III}	influence coefficient functions for Modes I, II and III
$\boldsymbol{F}_{max}$ $\boldsymbol{F}_{min}$	maximum and minimum influence coefficients of stress intensity factors
G, G_e	shear modulus and energy release rate
$J^1_{int}, J^2_{int}, J^3_{int}$	interaction J integral associated with auxiliary Modes I, II and III crack fields
J	J-integral
K_I, K_{II}, K_{III}	stress intensity factors for Mode I, II and III respectively
K_{eq}	equivalent stress intensity factor
l	half-length of a pipe
m	$=3-4\nu$ for plane strain, $=\frac{3-\nu}{1+\nu}$ for plane stress
M, N	defined terms for influence coefficient equations
n	unit vector perpendicular to the crack boundary
Q	elliptical integral of the second kind
r	radial distance from the crack tip
R, R_0	internal and external radius of a pipe
u_j	displacement components
β	eccentric anomaly for an ellipse ranging from 0 to 2 π
γ	crack propagation angle
δ	Kronecker delta
θ	inclination angle of a surface crack
κ	elastic strain energy density
λ	ratio of axial stress to internal pressure
ν	Poisson's ratio
ξ	position of a point along the semi-elliptical crack
σ	applied stress
σ_{ij}	stress components

corrosion propagation, high pH stress corrosion cracking (for steel pipes) [5], spiral welding technique [6] and bi/multi-axial stresses. As one of very few published papers on mixed mode fractures, Ayhan [7–8] obtained stress intensity factors for deflected and inclined corner and surface cracks under uniaxial tension by employing the enriched finite element method. However that work was limited to finite-thickness plates and only semi-circular surface cracks were considered. Li et al. [9] studied stress intensity factors for mixed mode fracture caused by inclined external cracks in metal pipes but their work was limited to internal pressure induced hoop stresses only.

For the research on stress intensity factors for cracks under axial stresses, induced by either axial tension or bending, current studies mainly concentrate on circumferential cracks. Raju and Newman [10] calculated stress intensity factors for external circumferential surface cracks in pipes and rods under both uniform tension and bending. Raju [11] and Mettu et al. [12] derived the stress intensity factors for a wide range of geometrical parameters of both the crack and pipe for different load distributions. Carpinteri and Brighenti [13] and Carpinteri et al. [14] examined the behaviours of part-through circumferential surface cracks in pipes under cyclic axial loading and bending respectively. Kou and Burdekin [15] studied stress intensity factors for circumferential deep surface cracks (crack depth to wall thickness ratio > 0.8), partly and fully through-wall cracks subjected to axial tension. Ghajar and Nabavi [16] determined stress intensity factors for internal circumferential cracks in pressurized cylinder subjected to axial stress induced by convection heat transfer on the inner surface.

A more comprehensive review of published literature [2–16] suggests that little attention has been drawn to the study on stress intensity factors for inclined cracks in metallic cylindrical structures and so far little research has been undertaken on the study of stress intensity factors for inclined cracks in pipes under axial tension and bending. As it is known, axial stresses can be caused by internal pressure if both ends of a pipe are enclosed, or restrained from movement due to pipe expansion and contraction [17]. Flexural bending of the pipe can be caused by inadequate bedding support or swelling and shrinkage of underlying clays which imposes additional axial tensile stresses. Therefore, there is a well-justified need to investigate the stress intensity factors for inclined cracks in pipes under axial stresses.

The aim of this paper is to determine the stress intensity factors for mixed mode fracture induced by inclined external surface cracks in pipes subjected to axial tension and bending. A combined J integral and the 3 dimensional finite element method developed in [9] is employed and a meshing technique with mixed types of quadratic tetrahedron and hexahedron elements is used to derive the stress intensity factors for mixed mode fracture. The propagation of inclined cracks in pipes is also investigated in the paper to predict the fracture failure of the pipe. After the proposed 3D finite element model is verified, the stress intensity factors for inclined cracks are obtained for various parameters of the crack and pipe, including different ratios of crack depth to half crack length (aspect ratios), crack depth to wall thickness ratios (relative depth ratios) and wall thickness to pipe internal radius ratios.

2. Stress intensity factors for mixed mode fracture

It is well known that, stress singularity exists at the crack front, the effect of which on the crack field is approximately proportional to $1/\sqrt{r}$ (r is the distance from the crack front). In order to quantify the stress singularity, stress intensity factor (SIF) $\boldsymbol{K}$ is introduced, which is a function of the applied loads and the geometry of both the crack and the structure where the crack occurs. The stress intensity factors are important not only for examination of brittle fracture but also for assessment of ductile fracture using the failure assessment diagram [18].

A cracked pipe can be subjected to a load that results in any of the three modes of fracture or a combination of two or all three modes. For cracked pipes, the formula for Mode I stress intensity factor at any location along an semi-elliptical crack in Raju and Newman [19] can be generalized for the stress intensity factors for mixed mode fracture as follows

$$\boldsymbol{K} = \sigma\sqrt{\frac{\pi a}{Q}}\boldsymbol{F}\left(\frac{a}{d}, \frac{a}{c}, \frac{d}{R}, \theta, \xi\right) \quad (1)$$

where $\boldsymbol{K} = \{K_I \quad K_{II} \quad K_{III}\}^{\boldsymbol{T}}$, K_I, K_{II} and K_{III} are stress intensity factors for Modes I, II and III respectively, σ is the applied stress, a is crack depth, Q is the shape factor for an ellipse and is given by the square of the complete elliptical integral of the second kind [20], d is the thickness of the pipe, c is half of crack length, R is