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Accurate and efficient analysis of stationary and propagating crack problems by meshless methods

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ABSTRACT

New numerical strategies based on meshless methods for the analysis of linear fracture mechanics problems with minimum computational labor are presented. Stationary as well as propagating cracks can be accurately modeled and analyzed by these proposed meshless techniques. For numerical analysis of the problem, meshless methods based on global weak-form are used. In order to capture the singular stress field near the crack tip, two different approaches are adopted. In the first approach, the asymptotic displacement fields are added to the basis functions of the meshless method. In the second one, a few nodes are added in the vicinity of the crack tip, while regular basis functions are used. The accuracy and stability of the two methods for determination of the stress intensity factors are then compared. In this work, an accurate integration technique, i.e., the background decomposition method (BDM), is utilized for efficient evaluation of the domain integrals of the weak-form with minimum computational cost. The superior accuracy of the proposed techniques is assessed by virtue of several benchmark problems. Through the presented numerical results it is concluded that the proposed methods are promising for the analysis of linear fracture mechanics problems.

1. Introduction

Cracks can be found in many structural components and machine parts and greatly affect the performance and life-time of the parts. Analysis of the stress field developed in a cracked body and investigation of the circumferences under which cracks propagate are one of the main subjects of fracture mechanics. Fracture mechanics has been long used for derivation of analytical formulas predicting the stress intensity factors (SIFs) in simple geometries and loading conditions. Early studies on cracked bodies are credited to Griffith [1] who developed a theory for the analysis of stress and displacement fields in a flat homogenous isotropic plate of uniform thickness, containing a straight crack. Since then, many other closed-form solutions are presented for determination of the SIFs of cracks, mostly in simple geometries and under simple loading conditions [2]. Nevertheless, obtaining analytical solutions of fracture mechanics problems in practical situations with complicated geometries and loading conditions is formidable. Consequently, accurate analysis of cracked bodies in practical circumstances lends itself to numerical methods [3].

Extensive research has been conducted on the area of numerical analysis of fracture mechanics and different techniques have been used successfully in this context. Finite element method (FEM) [4,5], boundary element method (BEM) [6,7], meshless methods [8–11], extended finite element method (XFEM) [12–14], numerical manifold method (NMM) [3], and the edge-based smoothed finite element method (ES-FEM) [15–24] are among the many numerical methods that have been used for the analysis of fracture mechanics problems. Since the present work focuses on the meshless techniques for analysis of stationary and propagating cracks, only a brief review of the literature on this subject is presented herein. Belytschko et al. [25] implemented the element free Galerkin (EFG) method for the analysis of problems of fracture and static crack growth. They later extended the application of the EFG method for the analysis of dynamic fracture problems [8]. Organ et al. [26] developed continuous meshless approximations for domains with non-convex boundaries, with emphasis on cracks. They developed two methods for obtaining smooth approximations

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of the field variable in such domains; a transparency method and a diffraction technique. They concluded that such remedies are only required when enriched basis functions are used in the problem formulation. Krysl and Belytschko [27] presented the EFG method for modeling of dynamically propagating cracks in three-dimensional (3D) bodies. In that work, the crack surface was defined by a set of triangular elements. Rao and Rahman [28] proposed a new weight function and a method for exact implementation of essential boundary conditions in the EFG method for analyzing linear-elastic cracked structures. Later, they extended their previous work for calculation of the stress intensity factors of stationary cracks in two-dimensional (2D) functionally graded materials (FGMs) of arbitrary geometry [29]. They also presented two interaction integrals for mixed-mode fracture analysis of FGMs. Duflot and Nguyen-Dang [30] made use of the EFG method for analysis of fatigue growth of cracks in 2D bodies using Paris' equation. Enriched weight functions were integrated into the formulation of their method to capture the singularity at the crack tip. The propagation of cracks in 3D bodies was investigated by Duflot with the use of meshless methods [31]. The stress singularity along the crack front was taken into account by an enrichment of the shape functions of the meshless method by means of appropriate weight functions. The proposed algorithms were used for prediction of non-planar propagation of multiple cracks. Wen et al. [32] investigated FGMs containing cracks by the EFG method in conjunction with enriched radial basis functions. Sladek et al. [33] proposed the use of meshless local Petrov–Galerkin (MLPG) method for stationary and transient crack analysis in 2D and 3D axisymmetric magneto-electric-elastic solids with continuously varying material properties. The complex variable meshless manifold method for fracture problems was presented by Gao and Cheng [34], based on the complex variable moving least-squares approximation and the finite cover theory. Zhuang et al. [35] mixed the ideas of visibility criterion and diffraction method to associate the displacement jump with the crack surface. Their formulation was based on the use of level set coordinates and the EFG method. They later extended their work to account for propagation of cracks in three dimensions [36]. Using the strain smoothing technique, Liu et al. [37] developed a cell-based radial point interpolation method (CS-RPIM) for analysis of fracture mechanics problems. Ghorashia et al. [38] investigated fracture analysis of orthotropic cracked media by applying the extended isogeometric analysis (XIGA) using the T-spline basis functions. Nguyen et al. [10] presented a new approach based on local partition of unity extended meshfree Galerkin method for modeling quasi-static crack growth in 2D elastic solids. They used the radial point interpolation method (RPIM) for generating the shape functions. In their work, representation of the crack topology was treated by the aid of the vector level set technique. Belytschko and Black [12] analyzed 2D dynamic stress concentration problems using the wavelet Galerkin method (WGM). They used a path independent J-integral to evaluate the dynamic stress intensity factor. A new meshless method based on Shepard function and Partition of Unity (MSPU) was proposed by Cai et al. [39] for calculation of crack SIFs and simulation of crack propagation. They made use of the virtual crack closure technique (VCCT) to capture the crack tip SIFs, and the crack propagation was determined based on the maximum circumferential stress criterion.

In all of the numerical methods for analysis of fracture mechanics problems the main focus is on accurate representation of the singular stress field near the crack tip. Classically, two different approaches have been adopted to model the singular behavior of the stress field at the crack tip. In the first approach, the basis functions of the meshless method are properly enriched to include a set of appropriate singular terms of the displacement and stress fields [9]. In the second one, a nodal refinement is carried out in a local sub-domain in the vicinity of the crack tip [40]. Fleming et al. [9] were among the first researchers who presented an enriched EFG formulation for the analysis of fracture problems. To obtain the enriched formulation, they proposed two different methods. In the first method, the asymptotic fields are added to the trial function, while the basis functions are augmented by the asymptotic fields in the second method. Irrespective of the approach used for capturing the singular stress field at the crack tip, numerical evaluation of the domain integrals in fracture mechanics problems requires a special treatment [9]. The common approach is to use a background mesh for evaluation of the domain integrals. Finer integration cells are usually used in the vicinity of the crack tip, and a high order Gaussian quadrature method is utilized in that cells. As reported by Fleming et al. [9], for a propagating crack, the discontinuities which arise in the shape functions and their derivatives when the crack passes through a quadrature cell are neglected. As the result, in the conventional integration methods, the discontinuity of the displacement filed along the crack line leads to numerical errors and instabilities, unless a large number of quadrature points are used in the problem domain.

In all of the aforementioned works, a somewhat complex procedure is utilized in order to model the singular behavior of the stress field near the crack tip. In this paper, a simple yet robust technique is proposed and it is shown that the crack tip SIFs and the crack propagation path can be handled accurately and with minimum burden. To this end, the background decomposition method (BDM) [41] is used for evaluation of the domain integrals of the weak-form. The BDM is a numerical domain integration technique which is specially designed for evaluation of integrals of the functions with severe variations in the problem domain. The BDM has been previously used for evaluating the domain integrals in meshless methods for the analysis of 2D and 3D elasto-statics problems with no cracks. Using the BDM, not only the severe variations of the integrand at the crack tip are grasped, but also the discontinuity of the shape functions and their derivatives are taken into account accurately. Consequently, this approach leads to very stable and accurate solutions for the stationary and propagating crack problems. The basic ideas of the present work were previously presented in a paper for modeling of stationary cracks [42]. Herein, these ideas are extended to the analysis of propagating cracks.

The proposed methodology of the present work can be implemented with any meshless method based on the weak formulation. Herein, the EFG method with enriched basis functions and also the RPIM with nodal refinement are adopted. The EFG was first proposed by Belytschko et al. for the analysis of heat conduction and thermo-elasticity problems [40]. Since then, the method has found many other uses and proved to be a robust technique for the analysis of many engineering problems. The RPIM is another meshless technique that has found widespread use in many branches of science and engineering. The meshless RPIM was originally introduced by Liu and Gu [43,44] for the analysis of solid mechanics problems and was later developed for the analysis of many other engineering problems, including fracture mechanics [37].

The rest of this paper is structured as follows: In Section 2, the background decomposition method for evaluation of the integrals associated with the weak-form of the fracture mechanics problems is presented in detail. In Section 3, some basic ideas for determination of the SIFs with the *J*-integral method and determination of the crack trajectory are reviewed. This section is followed by a concise review of the meshless formulation of the fracture mechanics, in Section 4. Several numerical examples are presented to assess the accuracy and efficiency of the proposed techniques in