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Quasi-static crack propagation simulation by an enhanced nodal gradient finite element with different enrichments

Zuoyi Kang [a], Tinh Quoc Bui [a,*], Takahiro Saitoh [b], Sohichi Hirose [a]

[a] Department of Civil and Environmental Engineering, Tokyo Institute of Technology, 2-12-1-W8-22, Ookayama, Meguro-ku, Tokyo 152-8552, Japan
[b] Division of Environmental Engineering Science, Faculty of Science and Technology, Gunma University, Kiryu, Japan

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ABSTRACT

The recently developed consecutive-interpolation local enriched partition-of-unity method based on 4-node quadrilateral element (XCQ4) is used to study quasi-static crack propagation in 2-dimensional solids. For some of these problems numerical results have also been calculated with the standard extended finite element (XQ4), provided that the two approaches have the same number of degrees of freedom. In addition, two different versions of enrichment functions capturing the crack tip fields are taken into account, integrating into either the XCQ4 or XQ4. In each case, results have been computed with both settings and compared between each other. It is found that the numerical solution using the XCQ4 element has better accuracy than that found with the XQ4, and these solutions agree well with reference solutions available in literature. The underlying difference between the consecutive-interpolation basis functions and those for the traditional XQ4 is that the former approximation functions constructed by incorporating both nodal values and averaged nodal gradients obtained from linear shape function as interpolation conditions, enhancing and smoothing the stress fields and stress intensity factors. Additionally, the conditioning issue of the developed method is also numerically examined.

1. Introduction

The classical finite element method (FEM) has been proven to be effective numerical methods for solving many engineering problems [1]. Particularly modelling crack propagation, however, remeshing is required in the FEM and that must be carried out during the evolution of crack. This task is cumbersome and it makes the method not effective in general. The extended finite element method (XFEM) and its variants, e.g., see Belytschko and Black [2], Zhang and Bui [3], Sharma et al. [4], Kumar et al. [5], Stroubou-lis et al. [6], Fries and Belytschko [7] have developed as its original goal is to overcome the drawbacks of the FEM. The partition of unity scheme [8] is utilized in XFEM to facilitate the standard approximation of displacement by a set of local enriched elements to accurately acquire the fracture parameters of discontinuity, which make modelling the moving discontinuity without altering the initial prescribed mesh. Meanwhile the set of Heaviside and asymptotic crack tip enrichment functions increase the power of local solution by incorporating the arbitrary functions into the basis of the FEM.

Alternatively, extended meshfree methods [9,10], which handle only the nodal data to describe the crack, allow representation of crack topology with the aid of the vector level set technique without the finite element mesh. Recently, the edge-based strain smoothing technique using a special singular element [11] or the extended isogeometric analysis in terms of local partition of unity method [12,13] have also been introduced and applied to deal with fracture problems in solids as well as in multiphase materials. Other approaches such as the Lepp-Delaunay based on mesh refinement algorithm for triangular meshes [14], and the scaled boundary finite element method (SBFEM) [15,16] are those that devoted to simulate crack propagation.

The new enrichment functions [17,18] for crack tip field are proposed as only two additional degrees of freedom are added at each support nodes of the tip element to reduce the matrix size. For solving the quasi-static and mixed mode fracture problem several numerical research achievements can be followed [19–24]. Further studies using the enrichments devoted to advanced composite materials and complicated configurations are also interesting to researchers majoring in computational fracture mechanics, see e.g., Bayesteh et al. [25], Bui and Zhang [26], Yu et al. [27],

* Corresponding author at: Department of Civil and Environmental Engineering, Tokyo Institute of Technology, 2-12-1-W8-22, Ookayama, Meguro-ku, Tokyo 152-8552, Japan.
E-mail address: tinh.buiquoc@gmail.com (T.Q. Bui).

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Nomenclature

$\mathbf{x} = (x, y)$	point of interest in 2D
$\widetilde{N}_f$	consecutive-interpolation shape functions
$N_f^{[i]}$	shape functions at node i
$\overline{N}_{f,x}^{[i]}$	Average derivatives of shape functions
e	element
d_f	nodal displacement
n_s	total number of supporting nodes
w_e	weight function of an element
Δ_e	area of the element
ϕ_i, ϕ_{ix}, ϕ_{iy}	polynomial basis
I^s	total nodes
J^{cut}	enriched nodes at crack faces
K^{tip}	enriched nodes at crack tip
$\mathbf{u}_i$	vector of nodal degrees of freedom
$H(\mathbf{x})$	discontinuous Heaviside function
$\mathbf{a}_i$	vector of enriched nodes
$F^{\alpha}(\mathbf{x})$	asymptotic crack-tip branch functions
$\mathbf{b}_k^{\alpha}$	vector of enriched nodes
$R(\mathbf{x})$	Ramp function
$\bar{J}(\mathbf{x})$	Heaviside-junction enrichment of crack
$f^n(\mathbf{x})$	signed distance function of the minor crack
$f^N(\mathbf{x})$	signed distance function of the main crack
(r, θ)	polar coordinate at crack-tip
(ξ, η)	natural coordinate system
E	Young's modulus
ν	Poisson's ratio
K_I, K_{II}	stress intensity factors
$q(\mathbf{x})$	smoothed function
τ	shear load
σ	tensile load
θ	crack angle
θ_c	crack growth orientation

Bui and Zhang [28], Bui et al. [29], Bhardwaj et al. [30], Yu et al. [31].

Recently, the so-called twice-interpolation finite element method (TFEM) using triangular elements is proposed by Zheng et al. [32] for solid mechanics problems. This new approach involves two stages of interpolation to construct the trial function. The first stage performs similar to that of the traditional FEM, however the average nodal gradients are additionally computed in the second stage to form the interpolation functions. Hence the trial function rebuilt by combining the nodal displacement and average gradients reveals more consecutive nodal gradients and higher order polynomial compared with the normal FEM. Later, the twice-interpolation method is further developed and employed in commercial ALOF by Wu et al. [33] for crack propagation in 2D elastic solids by coupling the nodal relaxation technique and node projection technique. Yang et al. [34] provided a three-node triangular element with continuous nodal stress. The numerical methodology done in Yang et al. completely differs from the twice-interpolation based element [33], as a result of applying the idea of the previous partition-of-unity based FE-Meshless quadrilateral element. Bui et al. [35] developed a new 4-node quadrilateral element with continuous nodal stress based on the twice-interpolation procedure (also known as consecutive-interpolation procedure – CIP) to the standard 4-node quadrilateral element for stress analysis of 2D elastic solids. Then, Kang et al. [36] further extended the CIP-based CQ4 element [35] to linear fracture problems in 2D by enrichment of the approximation functions in terms of local partition of unity method [8]. The method is named as XCQ4, which shows higher accuracy of the SIFs and smoother stresses than that computed using the classical XFEM [36]. The purpose of XCQ4 element is obvious from the aforementioned advantages and can be summarized concisely that the trial solution and its derivatives are continuous across inter-elements. This improvement could enhance the accuracy of the gradients of trial solution and also avoid handling smoothing operation technique often operated at the post-processing stage. Another important point should be noted that the proposed approach does not alter the total number of the degrees of freedom (DOFs), implying that both approaches take place the same total number of DOFs.

The motivation of this work is to extend the recently developed XCQ4 element to the modelling of crack propagation problems in 2D solids. Upon achievements in Bui et al. [35] and Kang et al. [36], it can be observed that the proposed XCQ4 performs well for the SIFs involving single and mixed-mode fracture problems, and that we expect to obtain good results in crack propagation which is being presented in this work. It is because the XCQ4 approximation functions not only well capture the discontinuity and singularity induced by cracks through the enrichments, but also improve the accuracy of the stresses, consequently the stress intensity factors (SIFs). In our work, the discontinuous Heaviside function is taken to treat the discontinuity cut by crack, while the asymptotic crack-tip branch functions are embedded into the approximation functions to capture the singular field at the crack tips. An alternative way of capturing the singular field at the crack tip, another version of enrichment function, the ramp function, along with the Heaviside function [17,18] is taken into account. We consider to integrate the ramp functions into both the standard XFEM using 4-node quadrilateral element (XQ4) and the developed XCQ4. The SIFs calculated by using both approaches, the ramp function with XCQ4 and with XQ4, are validated against reference solutions and can be found in the numerical examples.

To determine the direction of crack growth, as stated in Belytschko and Black [2] the maximum energy release rate criterion, the maximum circumferential stress criterion or the maximum principle stress criterion and the minimum strain energy density criterion can be used. In this work, the maximum circumferential stress criterion is taken, while the domain form of interaction integral is utilized for extracting the fracture parameters. The accuracy and performance of the proposed XCQ4 for modelling crack growth problems are demonstrated through six numerical examples of fracture in 2D. The single and mixed-mode fracture problems with complex configurations are considered. Additionally, the conditioning analysis of each approach is numerically examined.

The present paper is constructed in five sections. In Section 2, formulation of the proposed XCQ4 element for cracks is illustrated, in which the enriched approximation of displacement using the developed XCQ4 methodology and construction of the CQ4 shape functions and their characteristics, and the corresponding weak form of the governing equation are involved. The calculation of crack growth orientation based on the selected fracture criterion and modified integration path of interaction integral are presented in Section 3. The performances of the proposed XCQ4 for modelling crack evolution are estimated by implementing six numerical examples and the detailed discussions are subsequently stated in Section 4. The last section summarizes the key points appeared in this work and relevant constructive conclusions and remarks are represented.