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Effective local stress intensity factor criterion for prediction of crack growth trajectory under mixed mode fracture conditions

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ABSTRACT

To predict the crack propagation path under static and fatigue loading, a new criterion is proposed for general mixed mode I/II/III conditions. A review on the main brittle fracture criteria in the literature shows that most criteria predict a specific fracture angle in pure mode II without considering material properties, although the empirical results are considerably various for different materials. Moreover most of criteria predict a certain K_{IIIc}/K_{Ic} while the experiments show that the ratios of fracture toughness under pure mode III to that of mode I (K_{IIIc}/K_{Ic}) are considerably various for different materials. Since these matters are observed even in condition that the T stress term and differences in Poisson's ratios are negligible, thus in order to solve these issues, a material-dependent effective local stress intensity factor is proposed as a main fracture parameter. The ratios of in-plane and out of plane shear fracture strengths to tensile fracture strength are considered by defining two new material parameters. To verify the developed criterion, after characterization of the newly defined material parameters, the predicted crack extension directions and fracture locus have been compared with the available experiments at different mixed mode conditions. The predicted values are in a good agreement with the empirical ones for different materials although the proposed criterion uses the minimum number of experiments required to characterize the model.

1. Introduction

Loading conditions in cracked structures can be categorized into three fracture modes including mode I (opening mode), mode II (sliding mode) and mode III (tearing mode). In real engineering structures, loading usually is a combination of these modes. The behavior of cracked structures under mixed mode conditions is investigated using fracture criteria which determine crack growth direction and fracture load. Prediction of the trajectory of crack propagation can be made accurately if the crack growth angle is precisely specified, especially in structures which are under fatigue loading condition. In fatigue loading, crack growth rate and structure life are significantly dependent on the crack growth trajectory. Thus the precise estimation of the angle of crack extension is of great importance. The crack propagation phenomenon is caused by complex interactions at crack tip. Researchers use the well-known concepts of stress, strain fields, and energy to determine the conditions at which crack growth initiates and try to obtain the associated parameters. In each of these criteria, only one of these parameters is studied in an attempt to find the extremum values and pertaining critical directions at which crack growth occurs. In accordance with the studied parameter (stress, strain or energy), fracture criteria can be classified into three groups – stress-based, strain-based, and energy-based. The commonly used criteria will be described in the following sections.

1.1. Energy-based criteria

The first energy based criterion in fracture mechanics was proposed by Griffith [1]. The *Griffith criterion*, which is an energy-based method, can only be used in pure mode I condition. According to the *Griffith* criterion, fracture takes place when the stored energy in material reaches its surface energy. However, this criterion cannot be used to determine the crack growth angle. Hussain et al. [2] extended the Griffith criterion and proposed *G criterion* with considering the assumption that the crack propagates in the direction which the *Strain Energy Release Rate* (SERR) is maximum. Other researchers developed the maximum SERR criterion in mixed mode crack growth conditions [3,4]. Hosseini-Toudeshky and Mohammadi [5] carried out a computational procedure for calculating the SERR based on *Virtual Crack Closure Technique* (VCCT). The *Strain Energy Density criterion* (SED) was developed by Sih [6]. This criterion considers that crack propagation takes place in the

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direction with the minimum strain energy density. This criterion accounted for plane stress and plane strain problems. Gope et al. [7] presented a simple procedure based on SED criterion based on the concept of crack surface relative displacement. The SED criterion has been extended for U-noched and V-noched components [8–13]. Through modification of the SED criterion, Theocaris [14] proposed the *T criterion*. In this criterion by decomposing the strain energy density into two distortional and dilatational components, it is considered that crack propagates in the direction with the maximum dilatational strain energy density along the boundary between the elastic and plastic zone around the crack tip. Unlike the SED criterion, this approach did not rely on material properties or stress state. Yehia [15] and Khan and Khraisheh [16] by reviewing the T criterion, developed a modified criterion named *R criterion*. In their criterion, crack growth occurs if the plastic zone radius in direction of crack extension reaches its critical value. Mŕoz and Dolinski [17], presented another expression of the T criterion. They proposed that crack grows in the direction of minimum distortional energy along a path of constant dilatational energy around the crack tip. *Maximum Tangential Strain Energy Density criterion* (MTSED) is another energy-based criterion which was presented by Koo and Choy [18]. This criterion considers that crack propagates in the direction with the maximum tangential strain energy density. Ayatollahi and Saboori [19] developed the MTSED criterion to be applicable in presence of mode III conditions. However this criterion predicts a unique angle of fracture in each mixed mode ratio for all materials which have the same Poisson's ratio.

1.2. Strain-based criteria

As for strain-based criteria, Chang [20] proposed maximum tangential strain criterion. In this criterion, crack is considered to propagate when the tangential strain reaches its maximum value. Although the stress at the crack tip exceeds yield stress and the material shows plastic behavior in this region, tangential strain is determined by elastic behavior. Newman et al. [21] proposed the Crack Tip Displacement (CTD) criterion. CTD is the sum of Crack Tip Sliding Displacement (CTSD) and Crack Tip Opening Displacement (CTOD) vectors. In this criterion, crack was considered to propagate in the direction perpendicular to the CTD vector. The CTOA (Crack Tip Opening Angle) is another crack driving force parameter which is closely related to CTOD parameter [21].

1.3. Stress-based criteria

As for stress-based criteria, Eerdogan and Sih [22] proposed *Maximum Tangential Stress (MTS) criterion*. This criterion considers that the crack propagates along the direction where the tangential stress has its maximum amount in a specific distance from the crack tip. The predicted results of this criterion are in a proper agreements with empirical results of brittle materials under fatigue loading [23–25]. Due to its simplicity and agreement with micromechanical models, this criterion is one of the commonly used criteria. Furthermore, this method was developed and used for anisotropic materials by Buczek and Herakovich [26], Carloni and Nobile [27]. Moreover, this criterion was extended to be applicable for U-notched and V-notched specimens [28–30]. To overcome the shortcomings of the MTS criterion, Williams and Ewing [31], Maiti and Smith [32], Smith et al. [33], Aliha and Ayatollahi [34,35], Cheng et al. [36] and Mirsayar [37] took the nonsingular stress term, namely T stress term, into account and extended the MTS criterion into the *Generalized Maximum Tangential Stress* (GMTS) criterion. Moreover, Goldstein and Salganik [38] introduced the principle of local symmetry which assumes that crack propagates along the path where mode II stress intensity factor (K_{II}) vanishes. Also Matvienko [39] proposed maximum average tangential stress criterion which considers that a crack propagates in the direction with the maximum average tangential stress along a constant radius around the crack front. In order to consider the effects of other stress components in crack growth direction, Sajjadi et al. [40] proposed the effective stress criterion for mixed mode I/II. Schöllmann et al. [41] proposed that in a general triaxial stress state, crack propagates along the direction of maximum principal stress in a cylindrical domain around the crack front. For plane-stress, this criterion correlates to the MTS criterion [42]. Although this criterion could be applied for mixed modes I/II/III, it predicts a unique angle for a certain mixed mode ratio for all materials. Su and Cui [43] and Khan and Khraisheh [44], by using Von Mises as the yielding criterion, developed a modified maximum circumferential stress criterion considering the maximum hoop (tangential) stress at the boundary between plastic and elastic zones. Kong [45] proposed the maximum stress triaxiality criterion (M-criterion). In this criterion, it is considered that crack propagates along the direction which the ratio of hydrostatic stress to Von-Mises equivalent stress is maximum. Based on the third stress invariant, Papadopoulos [46] developed the Det criterion which is another stress-based criterion.

Bhadauria et al. [47] have been performed a comparison between some of the abovementioned fracture criteria. It should be noted that the numerical algorithm of crack growth for each of criteria can be based on *Extended Finite Element Method* (XFEM) or remeshing [48,49].

As it will be explained in detail in the next section, in one of each of the aforementioned criteria, without considering T stress term, a certain angle of crack extension would be found for pure mode II; however, the results of experiments on different materials vary. Moreover most of criteria predict a certain K_{IIIc}/K_{Ic} for different materials while the experiments show that the ratios of fracture toughness under pure mode III condition to that of mode I (K_{IIIc}/K_{Ic}) are considerably different for various materials. These matters can be due to the differences between the ratios of sliding and tearing strength to tensional strength of one material in comparison to another. The main purpose of present study is to develop a criterion based on considering these matters, to accurately predict the mixed-mode crack growth. This criterion is based on an effective local stress intensity factor as a fracture controlling parameter containing new material parameters for making proper consistency with the experimental results. Next, after characterization of material parameters, in order to evaluate the developed criterion, the predicted crack extension directions and fracture locus are compared with the available experiments at various mixed mode conditions.

2. A survey on the results of existing criteria

Using Irwin's equations in the polar coordinate system (r and θ) at crack tip, stress state around the crack front at the framework of linear elastic fracture mechanics (LEFM) can be expressed as follows [50] (see Fig. 1):

$$\begin{aligned}
\sigma_{rr} &= \frac{1}{\sqrt{2\pi r}}\left[\frac{K_I}{4}\left(5\cos\frac{\theta}{2}-\cos\frac{3\theta}{2}\right)+\frac{K_{II}}{4}\left(-5\sin\frac{\theta}{2}+3\sin\frac{3\theta}{2}\right)\right]+T\cos^2\theta \\
\sigma_{\theta\theta} &= \frac{1}{\sqrt{2\pi r}}\left[\frac{K_I}{4}\left(3\cos\frac{\theta}{2}+\cos\frac{3\theta}{2}\right)+\frac{K_{II}}{4}\left(-3\sin\frac{\theta}{2}-3\sin\frac{3\theta}{2}\right)\right]+T\sin^2\theta \\
\tau_{r\theta} &= \frac{1}{\sqrt{2\pi r}}\left[\frac{K_I}{4}\left(\sin\frac{\theta}{2}+\sin\frac{3\theta}{2}\right)+\frac{K_{II}}{4}\left(\cos\frac{\theta}{2}+3\cos\frac{3\theta}{2}\right)\right]-T\sin\theta\cdot\cos\theta
\end{aligned} \quad (1)$$

K_I and K_{II} are global stress intensity factors pertaining to pure mode I and II, respectively, and T is the nonsingular stress term so-called T stress term. At crack tip vicinity, the T term is ignorable in

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