Theoretical and Applied Fracture Mechanics xxx (2016) xxx–xxx

Contents lists available at ScienceDirect

Theoretical and Applied Fracture Mechanics

journal homepage: www.elsevier.com/locate/tafmec

Dynamic stress-intensity factors of an in-plane shear crack in saturated porous medium

P. Phurkhao

Department of Civil Engineering, Rangsit University, Pathumthani 12000, Thailand

ARTICLE INFO

Article history:
Received 22 March 2015
Revised 21 April 2016
Accepted 21 April 2016
Available online xxxx

Keywords:
Transient
SV-wave
Biot's theory
Porous medium
Dynamic stress-intensity factor

ABSTRACT

Based on the classical Biot theory of propagation of elastic waves in fluid-saturated porous media, two classes of plane problem are considered. One is the transient problem of a finite crack subjected to a suddenly applied in-plane shear, and another one is dealing with diffraction of transverse SV-wave by the same crack. The formulation utilizes the Laplace and integral transforms to reduce the mixed boundary-value problems to Fredholm integral equations of the second kind. The equations are then solved numerically for time and frequency dependent dynamic stress-intensity factors (mode II), and the influence of pore fluid on the dynamic stress-intensity factor is investigated. Pore fluid is found to have impact on the magnitude of the stress-intensity factor the extent of which depends on the two fluid parameter values, namely ratio of fluid mass with respect to the whole bulk mass and viscosity-to-permeability ratio. Comparisons of the solutions obtained from the present study with the corresponding known solutions of dry elastic medium are also provided to verify the validity of the present solution scheme.

1. Introduction

Fractures of elastic bodies by time-varying loads are often encountered when the bodies containing an internal flaw or crack are subjected to fluctuating loads and/or suddenly applied loads. Disturbances generated by such loads in the form of elastic waves are reflected and refracted at the crack and give rise to high local stress intensifications. This occurrence may eventually lead to a complete failure of the bodies by propagation of the crack once the magnitude of the singular stress at the crack tip which is measured in terms of the stress-intensity factor reaches a critical value.

In fracture mechanics, the state of stress near the crack region plays an important role in determining the stability of the crack, and one of the prime parameters used to predict initiation of the crack is the stress-intensity factor. For elastic bodies weakened by a flat circular crack or a line crack and subjected to dynamic loadings, past efforts have been concentrated mainly on elastic bodies of a single solid component [1–7]. However, some materials in engineering applications are porous in nature and possess an ability to absorb water or fluid and even allow fluid to flow through the interconnected pores. Medium made of such materials and containing a flaw or crack when subjected to dynamic loadings the stresses and displacements can be greatly different from those of the corresponding single-solid material. The theory that takes into account compressibility and viscosity of the fluid and leads to the coupling between fluid and solid was proposed by Biot [8,9]. This theory may be considered as an extension of his earlier version of the quasi-static theory [10]. According to the dynamical Biot theory of elastic waves propagation in fluid-saturated porous media, two types of dilatational wave (P-wave), namely P-wave of the first kind (or P1-wave) and P-wave of the second kind (or P2-wave), may propagate through the medium besides the shear wave (S-wave). Several contributions related to time-dependent crack problems have been made based on quasi-static theory [11–14]. The dynamics of permeable and impermeable semi-infinite cracks propagating in a fluid-saturated medium have been studied by Loret and Radi [15] based on the field equations of Biot [8] and Bowen [16]. Another version of the dynamical Biot theory [9] was applied by Jin and Zhong [17] to investigate the influence of pore fluid on the dynamic stress-intensity factor (mode I) for the transient problem of a penny-shaped crack subjected to suddenly applied normal loadings. A recent investigation on diffraction of P1-wave by a penny-shaped crack in porous medium was made by Galvin and Gurevich [18] who applied the Hankel transform to reduce the problem to the Fredholm integral equation and obtained the far-field solution in terms of the scattering cross-section. This information is found to be useful in quantitative nondestructive evaluation. However, it still lacks the detailed information of stresses at the crack region which is necessary to assess the stability of the crack. Further progress in this class of

E-mail address: pichaya@rsu.ac.th

http://dx.doi.org/10.1016/j.tafmec.2016.04.007
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problem was made by Phurkhao [19], who successfully solved a similar problem by assuming the faces of the same crack are impermeable and subjected to the incident P1 and/or P2 waves. In connection with this problem the near-field and far-field solutions and the dynamic stress-intensity factors are presented in the report. The same approach was also used in his earlier paper [20] to solve a 2D plane strain problem of a line crack with finite width in the porous medium.

Just as oscillating compressional force produces dilatational wave (P-wave), fluctuating shear force generates disturbances in the form of shear wave (S-wave). Generally, in the propagation of plane waves, the displacement vector which describes the motion of the P-wave is always parallel to the direction of wave propagation, while the S-wave which moves at a slower speed its transverse motion lies in the plane normal to the direction of propagation. The S-wave can be polarized parallel to the vertical plane referred to as SV-wave and the horizontal plane referred to as SH-wave. In this case their displacement vectors are normal to each other. At geometric discontinuities or cracks the incidence of either P or SV-wave can cause scattering of waves and generates scattered waves in the form of both P and SV waves. Such phenomenon when occurs in the porous medium saturated with fluid will be much more complicated. This is due to the fact that the scattered P-wave will propagate not only in the solid part (P1-wave) but also in the fluid component (P2-wave) as well. This will further add more mathematical complexity in solving the problem. An earlier investigation related to shear fracture problem of a semi-infinite crack within the porous medium which is subjected to impulsive loadings has been made by Craster and Atkinson [14]. However, their formulation relies on the assumptions made in the quasi-static Biot theory. As for the scattering of SV-wave by a finite crack within the porous medium, to the best of the author's knowledge, there is still a lack of analytical solutions needed to provide a physical insight into the intrinsic influence of the pore fluid on the intensity of the local stress field around the crack region.

The objective of this study is to develop an approach to determine the response of a saturated porous medium containing a stationary and permeable crack of finite width subjected to time-dependent loadings. Two loading cases are considered separately. One is an in-plane shear applied suddenly to the crack surfaces, and another one is the time-harmonic SV-wave impinging on the same crack. Of particular interest is the influence of the pore fluid on the behavior of the singular stress field in the vicinity of the crack tips as well as the extent and how the dynamic stress-intensity factor is affected by the presence of the fluid when the dynamical theory of Biot is employed. It is hopeful that any knowledge emerging from this investigation may lead to a more accurate and reliable prediction of fracture failure in fluid-infiltrated porous medium.

A brief summary of the general field equations governing the motion of solid and fluid in the porous medium is given in Section 2. Formulation and solution in the Laplace transform domain for the transient problem of an in-plane shear crack are presented in Section 3. Here, the numerical results obtained through the numerical Laplace inversion [21] in terms of the mode-II dynamic stress-intensity factor in the usual time domain are illustrated graphically. The scattering of SV-wave by crack is considered in Section 4. Relevant expressions of incident field in the absence of crack in the medium are derived. The shearing stress of the same magnitude but opposite in direction to that developed for the incident wave is then applied to the faces of the crack to generate the scattered wave field. The integral transform solution is derived, and the mixed boundary conditions are applied to reduce the problem to a Fredholm integral equation of the second kind. As an application of the present solution scheme, numerical results calculated in terms of the mode-II stress-intensity factors for some selected material properties are presented to indicate the influence of pore fluid. Accuracy of the present solution procedure is also demonstrated by comparing the present solution in the limiting case of dry medium with the known solution in literature. Finally, conclusions drawn from the numerical results are presented in Section 5.

2. Basic equations

With reference to a rectangular Cartesian coordinate system (x, y, z) where the space variables x, y, z are normalized by a reference dimension a (which is half of the crack width) and the time variable t by a/V_s with V_s being the shear wave velocity of the medium, the non-varnishing components of displacements (non-dimensionalized by a) in a saturated porous medium under the state of plane strain parallel to the xy-plane can be represented in terms of the potentials $\phi(x, y, t)$, $\psi(x, y, t)$, $\phi_f(x, y, t)$ and $\psi_f(x, y, t)$ as

$$u_x = \partial\phi/\partial x + \partial\psi/\partial y, \quad u_y = \partial\phi/\partial y - \partial\psi/\partial x, \tag{1}$$

$$w_x = \partial\phi_f/\partial x + \partial\psi_f/\partial y, \quad w_y = \partial\phi_f/\partial y - \partial\psi_f/\partial x. \tag{2}$$

In Eqs. (1) and (2), u_x and u_y represent the displacement components of the solid, while $w_x = f(U_x - u_x)$ and $w_y = f(U_y - u_y)$ stand for the relative displacement components of the fluid with respect to the solid portion. Here, U_x and U_y represent the components of fluid displacement vector, and f denotes the porosity of the medium.

Assuming no body forces, the non-dimensional equations governing the wave propagation in the porous medium can be described in terms of the scalar potentials as given by Phurkhao [20]. They are briefly summarized here for the sake of easy reference.

$$\beta^2\nabla^2\phi + \alpha\alpha_f\nabla^2\phi_f = \frac{\partial^2\phi}{\partial t^2} + q\frac{\partial^2\phi_f}{\partial t^2}, \tag{3}$$

$$\alpha_f\nabla^2(\alpha\phi + \phi_f) = q\frac{\partial^2\phi}{\partial t^2} + \frac{q}{f}\frac{\partial^2\phi_f}{\partial t^2} + b\frac{\partial\phi_f}{\partial t}, \tag{4}$$

$$\nabla^2\psi = \frac{\partial^2\psi}{\partial t^2} + q\frac{\partial^2\psi_f}{\partial t^2}, \tag{5}$$

$$q\frac{\partial^2\psi}{\partial t^2} + \frac{q}{f}\frac{\partial^2\psi_f}{\partial t^2} + b\frac{\partial\psi_f}{\partial t} = 0. \tag{6}$$

Here, the parameters q and b denote, respectively, the ratio of the fluid mass density to the whole bulk mass (ρ_f/ρ) and the dimensionless viscosity-to-permeability ratio $(\eta a/k\rho V_s)$. The constant α represents the Biot coefficient and $\alpha_f = M/\mu$ the Biot modulus non-dimensionalized by the shear modulus of the bulk material. The constant $\beta = V_c/V_s$ designates the ratio of the reference velocities of the dilatational wave $V_c = [(\lambda_u + 2\mu)/\rho]^{1/2}$ and the shear wave $V_s = (\mu/\rho)^{1/2}$ under the "dynamic compatibility" condition with $\lambda_u = \lambda + \alpha^2 M$ and λ being Lame's constant of the bulk material under undrained and drained conditions, respectively.

Eqs. (3) and (4) are equations governing the propagation of dilatational waves, while Eqs. (5) and (6) govern the shear waves. Additionally, some of the dimensionless stress components and pore pressure (non-dimensionalized by μ) relevant to the subsequent analysis are

$$\tau_{yy} = 2\left[\frac{\partial^2\phi}{\partial y^2} - \frac{\partial^2\psi}{\partial y\partial x}\right] + (\beta^2 - 2)\nabla^2\phi + \alpha\alpha_f\nabla^2\phi_f, \tag{7}$$

Please cite this article in press as: P. Phurkhao, Dynamic stress-intensity factors of an in-plane shear crack in saturated porous medium, Theor. Appl. Fract. Mech. (2016), http://dx.doi.org/10.1016/j.tafmec.2016.04.007