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Determination of fracture parameters in center cracked circular discs of concrete under diametral loading: A numerical analysis and experimental results

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ABSTRACT

In this paper, a unified approach combining the interaction integral method with the extended finite element method is proposed to evaluate the mixed-mode stress intensity factors (SIFs) and T-stress for center cracked circular discs (CCCD) of concrete. In order to predict the crack initiation angles and fracture resistances, the generalized maximum tangential stress (GMTS) criterion is employed, which simultaneously involves the effects of the mixed-mode SIFs, the T-stress and a physical length scale r_c (the size of the fracture process zone, FPZ). Moreover, a series of mixed-mode fracture experiments are conducted in the full range from pure mode I to pure mode II using the concrete CCCD specimens. The dimensionless SIFs and T-stress, calculated by the numerical method, are firstly compared with the available results in the literature to verify its accuracy and practicability. By incorporating the above dimensionless fracture parameters, a simple strength based hypothesis, the GMTS criterion and the mode I fracture resistance obtained by the diametral compression tests, the mixed-mode crack initiation angles and fracture resistances are predicted. Finally, the predicted results are compared with the experimental results. It is shown that both the fracture resistances and the crack initiation angles predicted by the GMTS criterion are more close to the experimental results than those predicted by the traditional maximum tangential stress (MTS) criterion.

1. Introduction

Nowadays, concrete is still one of the most widely used structural strength materials in civil engineering and other industry fields [1]. As everyone knows, enormous attention has been paid for the fracture behavior of concrete based on experimental, theoretical and numerical methods [2–4]. Due to the variety of ingredients and numerous defects in the process of production, concrete has relatively complex internal structure and shows the dispersion to some degree, leading to amount of difficulties for probing their fracture mechanisms. During the numerous studies, how to effectively obtain the fracture toughness of concrete, especially for the case of mixed-mode loading condition, is an eternal topic.

Since concrete and rock have similar mechanical properties, the test methods on rock can be introduced to evaluate the mechanical properties of concrete. The Brazilian disc test, an indirect test method of tensile strength used for brittle materials such as concrete, rock and rock-like, draws people's attention again recently [5]. The CCCD specimens can be conveniently used to investigate mixed-mode brittle fracture behavior by changing the angle between precast crack direction and the loading direction [6–9]. Meanwhile, there are several criteria for predicting the onset of mixed-mode fracture in brittle materials based on the pure mode I fracture resistance [10]. The maximum tangential stress criterion [11], the maximum energy release rate criterion [12] and the minimum strain energy density criterion [13] are to name a few. However, these criteria could not able to accurately predict the onset of mixed-mode fracture for any given rock or concrete materials. In light of the advantages of simple form and higher accuracy, a generalized maximum tangential stress (GMTS) criterion [14,15] has been used to evaluate the crack initiation angles and fracture resistances of CCCD specimens, which takes into account the effect of

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the T-stress besides the SIFs. The T-stress or the first non-singular stress term of the Williams expansion, which acts parallel to the crack line, plays a significant effect on the crack behavior, especially for the fracture toughness in the case of mode I and mixed-mode cracks [16,17]. In addition, GMTS criterion could also consider the effect of geometry of the CCCD specimen, for example, the half crack length a and the diameter of disc d [18].

In order to use the GMTS criterion, the dimensionless parameters of SIFs and T-stress at the crack tips should be determined firstly. Ayatollahi et al. [19] proposed a finite element over-deterministic (FEOD) method to calculate the coefficients of Williams expansion. However, the finite element based method could be a more time consuming choice because that the dimensionless parameters of full range from pure mode I to pure mode II for CCCD specimens need a large number of meshing and calculating processes. As a result, the extended finite element based method could be more competitive herein due to the fact that the XFEM allows discontinuous boundaries, such as cracks or material interfaces, to be independent of the mesh [20–24]. Wang et al. [22–24] proposed a local mesh replacement method based the XFEM for the modeling of crack growth in the materials containing different kinds of interfaces. The method does not contain either material constitutive relations or the crack-tip enriched functions and thus has a lager applicable scale than the traditional XFEM. On the other hand, the interaction integral method is widely used to evaluate the mixed-mode SIFs and T-stress in homogeneous and non-homogeneous materials [20,21,25–28]. For example, a unified approach combining the interaction integral method and the finite element method (FEM) was provided by Kim et al. [26] to evaluate both mixed-mode SIFs and T-stress for functionally graded materials with general material properties including either continuum functions or micro mechanics model. Yu et al. [20,25,27,28] developed another interaction integral method for solving the SIFs effectively in materials with complex interfaces. They proved that the new derived interaction integral method does not contain any derivatives related to material properties and is valid even when the integral domain contains material interfaces. However, the interaction integral given by Yu et al. [20,25,27,28] has not been unified so far for solving both the SIFs and T-stress and to the best of the author's knowledge, few works have been done based on the XFEM and the interaction integral method for investigating the fracture behavior of CCCD specimen.

The outline of this paper is as follows. Section 2 elaborates the interaction integral and the applicable auxiliary fields. A local mesh replacement method based on the XFEM is introduced in Section 3. Section 4 presents computations of the mixed-mode SIFs, T-stress, crack initiation angles and fracture resistances. In Section 5, two groups of concrete CCCD specimens are tested under quasi-static diametral compression loading. Comparisons and discussions of the numerical and the experimental results are performed in Section 6. Finally, Section 7 concludes this work.

2. The interaction integral

The interaction integral method, firstly proposed by Yau et al. [29], can be used to evaluate both the SIFs and T-stress by appropriately selecting the auxiliary fields. Two different auxiliary fields, originally developed for homogeneous and functionally graded materials, are chosen in present work for evaluating the SIFs and T-stress, respectively. Yu et al. [25,27] has proved that these auxiliary fields can also be applied for non-homogeneous materials with complex interfaces. An 'incompatible formulation' is used in the interaction integral, which accounts for the displacement mismatch between homogeneous and non-homogeneous materials [26]. These auxiliary fields selected in this paper are described as follows.

2.1. Auxiliary fields for SIFs

These auxiliary stress and displacement fields are chosen for evaluating mixed-mode SIFs according to the Williams' [30] crack-tip asymptotic stress and displacement fields, respectively [26]. A two dimensional (2D) crack in local Cartesion and polar coordinates is shown in Fig. 1(a), and the auxiliary fields are chosen as below:

$$\sigma_{ij}^{aux} = \frac{K_I^{aux}}{\sqrt{2\pi r}} f_{ij}^{I}(\theta) + \frac{K_{II}^{aux}}{\sqrt{2\pi r}} f_{ij}^{II}(\theta), \quad (i,j = 1,2) \tag{1}$$

$$u_i^{aux} = \frac{K_I^{aux}}{\mu_{tip}} \sqrt{\frac{r}{2\pi}} g_i^{I}(\theta) + \frac{K_{II}^{aux}}{\mu_{tip}} \sqrt{\frac{r}{2\pi}} g_i^{II}(\theta), \quad (i = 1,2) \tag{2}$$

where μ_{tip} is the shear modulus at the crack tip, and K_I^{aux} and K_{II}^{aux} are the auxiliary mode I and II SIFs, respectively. The angular functions $f_{ij}(\theta)$ and $g_i(\theta)$ are given by the following forms, respectively [31].

$$\begin{aligned} &f_{11}^{I}(\theta) = \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right), \quad f_{22}^{I}(\theta) = \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \\ &f_{11}^{II}(\theta) = -\sin\frac{\theta}{2}\left(2 + \cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right), \quad f_{22}^{II}(\theta) = \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2} \\ &f_{12}^{I}(\theta) = f_{21}^{I}(\theta) = f_{22}^{II}(\theta), \quad f_{12}^{II}(\theta) = f_{21}^{II}(\theta) = f_{11}^{I}(\theta) \end{aligned} \tag{3}$$

$$\begin{aligned} g_1^{I}(\theta) &= \frac{1}{2}\cos\frac{\theta}{2}\left(k_{tip} - 1 + 2\sin^2\frac{\theta}{2}\right), \\ g_2^{I}(\theta) &= \frac{1}{2}\sin\frac{\theta}{2}\left(k_{tip} + 1 - 2\cos^2\frac{\theta}{2}\right) \\ g_1^{II}(\theta) &= \frac{1}{2}\sin\frac{\theta}{2}\left(k_{tip} + 1 + 2\cos^2\frac{\theta}{2}\right), \\ g_2^{II}(\theta) &= -\frac{1}{2}\cos\frac{\theta}{2}\left(k_{tip} - 1 - 2\sin^2\frac{\theta}{2}\right) \end{aligned} \tag{4}$$

where k_{tip} is the Kolosov constant at the crack tip, and k_{tip} equals $(3 - \upsilon_{tip})/(1 + \upsilon_{tip})$ for generalized plane stress condition and $(3 - 4\upsilon_{tip})$ for plane strain condition, υ_{tip} is the Poisson's ratio at the crack tip.

2.2. Auxiliary fields for T-stress

These auxiliary stress and displacement fields are selected for evaluating T-stress according to a point force F, which is applied to the crack tip in the x_1 direction in an infinite plane as shown in Fig. 1(b). The auxiliary fields are described as follows:

$$\begin{aligned} &\sigma_{11}^{aux} = -\frac{F}{\pi r}\cos^3\theta, \quad \sigma_{22}^{aux} = -\frac{F}{\pi r}\cos\theta\sin^2\theta, \\ &\sigma_{12}^{aux} = \sigma_{21}^{aux} = -\frac{F}{\pi r}\cos^2\theta\sin\theta \end{aligned} \tag{5}$$

$$\begin{aligned} u_1^{aux} &= -\frac{F(k_{tip}+1)}{8\pi\mu_{tip}}\ln\frac{r}{d} - \frac{F}{4\pi\mu_{tip}}\sin^2\theta, \\ u_2^{aux} &= -\frac{F(k_{tip}-1)}{8\pi\mu_{tip}}\theta + \frac{F}{4\pi\mu_{tip}}\sin\theta\cos\theta \end{aligned} \tag{6}$$

where r and θ are the polar coordinates, d is the distance between the coordinate origin and a fixed point on x_1 axis as shown in Fig. 1(b).

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