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Half-layers with interface cracks under anti-plane impact

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ABSTRACT

The solution to a dynamic screw dislocation situated at the interface of a half-layer composed of an isotropic substrate with orthotropic coating is obtained by means of the image method. The structural energy dissipation is modeled by viscous damping. Moreover, stress field in the intact layer, under a pair of anti-plane self-equilibrating point force is determined. These solutions are used to construct integral equations in the half-layers delaminated by edge and embedded cracks, subjected to dynamic anti-plane excitation. The integral equations are solved numerically for the density of screw dislocation on a crack surface and the results are used to determine stress intensity factor for cracks. The edge effect as well as interaction between two interface cracks is studied.

1. Introduction

The nucleation and subsequent growth of interfacial defects in layered structures may result in their catastrophic failure. Interfacial cracks subject to anti-plane deformation have been addressed by several investigators. A crack at the interface of two equal-sized dissimilar rectangular regions under tearing mode was the subject of study by Zhang [1]. Analysis of a single and also an array of periodic cracks between two dissimilar half-planes under oblique harmonic SH-waves were taken up by Zhang [2,3]. Jin and Batra [4] analyzed an interface crack between a functionally graded coating and an isotropic substrate subjected to static anti-plane shear. Wang and Gross [5] were concerned with the interaction of harmonic anti-plane waves with an array of periodic interface cracks in a multi-layered medium. They investigated the effects of a combination of layers' material properties, cracks geometry, wave frequency and its incident angle on the dynamic stress intensity factor (DSIF). Closed form solution for mode III deformation of a crack at the interface of two dissimilar elastic layers of equal thickness was derived by Li [6]. Ding and Li [7] analyzed anti-plane deformation of a single and also a periodic array of cracks at the interface of a functionally graded and a homogenous layer. The solution to an interface crack between two anisotropic half-planes under anti-plane time harmonic excitation was obtained by Bostrom and Golub [8]. Bostrom and Kvasha [9] were concerned with the propagation of time-harmonic SH-waves in a symmetric structure consisted of three anisotropic layers weakened by an array of periodic interface cracks. The solution to mode III fracture of two dissimilar infinite isotropic layers bonded to a functionally graded strip with two offset interfacial cracks under impact loads was carried out by Choi [10]. In another article, Choi [11] solved the problem of an interfacial edge and also an embedded crack in a half-layer composed of functionally graded and isotropic ones under static anti-plane deformation.

Anti-plane transient analysis of interfacial cracks between orthotropic and isotropic half-layers is taken up in this study. The method of images is employed to obtain the solution to a screw dislocation located at the interface of the layers. The stress components are utilized to construct integral equations for interfacial cracks. Integral equations are solved numerically to determine the density of dislocation on a crack surface thereby obtaining stress intensity factor at a crack tip. Several examples of edge and embedded cracks are solved and interaction between cracks is studied.

2. Formulation

We consider an isotropic layer with thickness h reinforced by an orthotropic layer having thickness h_1. The Cartesian coordinates are chosen such that the x-axis coincides with the interface; thus, $|x| < \infty, \ -h < y < h_1$. Moreover, the x-axis is in the direction of principal material properties of the orthotropic layer. The non-vanishing constitutive equations for the orthotropic layer with shear material constants C_{44} and C_{55} undergoing anti-plane deformation are

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$$\sigma_{xz} = C_{55}\frac{\partial w}{\partial x}$$
$$\sigma_{yz} = C_{44}\frac{\partial w}{\partial y}, \quad 0 < y < h_1 \tag{1}$$

And for the isotropic layer with elastic shear modulus μ, we have

$$\sigma_{xz} = \mu\frac{\partial w}{\partial x}$$
$$\sigma_{yz} = \mu\frac{\partial w}{\partial y}, \quad -h < y < 0 \tag{2}$$

Substitution of Eq. (1) into equation of motion, $\sigma_{ij,j} = \rho_l u_{i,tt} + Cu_{i,t}$, where ρ_l is the mass density of the orthotropic layer and C is the structural damping coefficient, yields

$$C_{55}\frac{\partial^2 w}{\partial x^2} + C_{44}\frac{\partial^2 w}{\partial y^2} = \rho_l\frac{\partial^2 w}{\partial t^2} + C\frac{\partial w}{\partial t} \tag{3}$$

Similarly, equation of motion for isotropic layer, leads to

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = C_s^2\frac{\partial^2 w}{\partial t^2} + \frac{C}{\mu}\frac{\partial w}{\partial t} \tag{4}$$

where $C_s = \sqrt{\rho/\mu}$ is the reciprocal of the shear wave velocity in the isotropic material. Let a screw dislocation with Burgers vector $B_z(t)$ be situated at the origin of coordinate system with dislocation cut $x > 0$. Screw dislocation is identified by

$$w(x, 0^+) - w(x, 0^-) = B_z(t)H(x)$$
$$\sigma_{yz}(x, 0^+) = \sigma_{yz}(x, 0^-) \tag{5}$$

where $H(\cdot)$ is the Heaviside step function. Moreover, the traction free boundary condition on the layer boundary reads as

$$\sigma_{yz}(x, h_1) = \sigma_{yz}(x, -h) = 0 \tag{6}$$

Application of the Laplace transformation to Eqs. (3) and (4), assuming stationary situation at the outset results in

$$C_{55}\frac{\partial^2 \bar{w}}{\partial x^2} + C_{44}\frac{\partial^2 \bar{w}}{\partial y^2} - s(\rho_l s + C)\bar{w} = 0, \qquad 0 < y < h_1$$
$$\frac{\partial^2 \bar{w}}{\partial x^2} + \frac{\partial^2 \bar{w}}{\partial y^2} - s(C_s^2 s + C/\mu)\bar{w} = 0, \qquad -h < y < 0 \tag{7}$$

where $\bar{w}(x, y, s) = \mathcal{L}[w(x, y, t);\ s]$. We further use a complex Fourier transform, $\mathcal{F}$, to eliminate variable x, arriving at

$$C_{44}\frac{d^2 W}{dy^2} - [s(\rho_l s + C) + C_{55}\alpha^2]W = 0, \qquad 0 < y < h_1$$
$$\frac{d^2 W}{dy^2} - [s(C_s^2 s + C/\mu) + \alpha^2]W = 0, \qquad -h < y < 0 \tag{8}$$

where $W(\alpha, y, s) = \mathcal{F}[\bar{w}(x, y, s);\ \alpha]$. The solutions to Eq. (8) are

$$W_k(\alpha, y, s) = A_{1k}\sinh(\lambda_k y) + A_{2k}\cosh(\lambda_k y), \qquad k \in \{1, 2\} \tag{9}$$

In Eq. (9) subscript $k \in \{1, 2\}$ designate, respectively, regions $0 < y < h_1$ and $-h < y < 0$ and

$$\lambda_1 = \sqrt{[s(\rho_l s + C) + C_{55}\alpha^2]/C_{44}},$$
$$\lambda_2 = \sqrt{s(C_s^2 s + C/\mu) + \alpha^2} \tag{10}$$

Utilizing Eqs. (5) and (6), we may derive necessary conditions for the determination of the unknown coefficients in Eq. (9) as

$$W_1(\alpha, 0, s) - W_2(\alpha, 0, s) = B_z(s)\left[\frac{i}{\alpha} + \pi\delta(\alpha)\right]$$
$$C_{44}\frac{dW_1}{dy}(\alpha, 0, s) = \mu\frac{dW_2}{dy}(\alpha, 0, s)$$
$$\frac{dW_1}{dy}(\alpha, h_1, s) = 0$$
$$\frac{dW_2}{dy}(\alpha, -h, s) = 0 \tag{11}$$

where $B_z(s) = \mathcal{L}[B_z(t);\ s]$, $i = \sqrt{-1}$, and $\delta(\cdot)$ is the Dirac delta function. Applying boundary conditions (11) to Eq. (9) and taking the inverse Fourier transform of resultant equations yields the Laplace transform of the displacement field

$$\bar{w}(x, y, s) = \frac{\mu}{2}\frac{\beta_2\sinh(\beta_2 h)B_z(s)[-\sinh(\beta_1 h_1)\sinh(\beta_1 y) + \cosh(\beta_1 h_1)\cosh(\beta_1 y)]}{C_{44}\beta_1\cosh(\beta_2 h)\sinh(\beta_1 h_1) + \mu\beta_2\sinh(\beta_2 h)\cosh(\beta_1 h_1)}$$
$$+\frac{\mu}{\pi}\int_0^\infty \frac{\lambda_2\sinh(\lambda_2 h)B_z(s)[\sinh(\lambda_1 h_1)\sinh(\lambda_1 y) - \cosh(\lambda_1 h_1)\cosh(\lambda_1 y)]\sin(x\alpha)}{\alpha[C_{44}\lambda_1\cosh(\lambda_2 h)\sinh(\lambda_1 h_1) + \lambda_2\mu\sinh(\lambda_2 h)\cosh(\lambda_1 h_1)]}d\alpha, \quad 0 < y < h_1$$
$$\bar{w}(x, y, s) = -\frac{C_{44}}{2}\frac{\sinh(\beta_1 h_1)\beta_1 B_z(s)[\sinh(\beta_2 h)\sinh(\beta_2 y) + \cosh(\beta_2 h)\cosh(\beta_2 y)]}{\cosh(\beta_2 h)C_{44}\sinh(\beta_1 h_1)\beta_1 + \sinh(\beta_2 h)\beta_2\cosh(\beta_1 h_1)\mu}$$
$$+\frac{C_{44}}{\pi}\int_0^\infty \frac{\lambda_1\sinh(\lambda_1 h_1)B_z(s)[\sinh(\lambda_2 h)\sinh(\lambda_2 y) + \cosh(\lambda_2 h)\cosh(\lambda_2 y)]\sin(x\alpha)}{\alpha[(\cosh(\lambda_2 h)C_{44}\sinh(\lambda_1 h_1)\lambda_1 + \sinh(\lambda_2 h)\lambda_2\cosh(\lambda_1 h_1)\mu]}d\alpha, \quad -h < y < 0 \tag{12}$$

where

$$\beta_1 = \sqrt{\frac{s(s\rho_l + C)}{C_{44}}}, \quad \beta_2 = \sqrt{\frac{s(C_s^2\mu s + C)}{\mu}} \tag{13}$$

From Eqs. (12), (1) and (2), the Laplace transform of stress components for a screw dislocation situated on the interface at $x = \xi$ become

$$\sigma_{xz}(x, y, s) = -\frac{\mu C_{55}B_z(s)}{\pi}\int_0^\infty \frac{\lambda_2\sinh(\lambda_2 h)[\cosh(\lambda_1(h_1 - y))]\cos[\alpha(x - \xi)]}{\lambda_1 C_{44}\cosh(\lambda_2 h)\sinh(\lambda_1 h_1) + \mu\lambda_2\sinh(\lambda_2 h)\cosh(\lambda_1 h_1)}d\alpha$$

$$\sigma_{yz}(x, y, s) = -\frac{\mu C_{44}B_z(s)}{2}\frac{\beta_1\beta_2[\sinh(\beta_1(h_1 - y))]\sinh(\lambda_2 h)}{C_{44}\beta_1\cosh(\beta_2 h)\sinh(\beta_1 h_1) + \mu\beta_2\cosh(\beta_1 h_1)\sinh(\beta_2 h)}$$
$$+\frac{\mu C_{44}B_z(s)}{\pi}\int_0^\infty \frac{\lambda_1\lambda_2\sinh(\lambda_2 h)[\sinh(\lambda_1(h_1 - y))]\sin[\alpha(x - \xi)]}{\alpha[C_{44}\lambda_1\cosh(\lambda_2 h)\sinh(\lambda_1 h_1) + \mu\lambda_2\sinh(\lambda_2 h)\cosh(\lambda_1 h_1)]}d\alpha, \quad 0 < y < h_1$$

$$\sigma_{xz}(x, y, s) = \frac{\mu C_{55}B_z(s)}{\pi}\int_0^\infty \frac{\lambda_1\sinh(\lambda_1 h_1)[\cosh(\lambda_2(y + h))]\cos[\alpha(x - \xi)]}{C_{44}\lambda_1\cosh(\lambda_2 h)\sinh(\lambda_1 h_1) + \mu\lambda_2\sinh(\lambda_2 h)\cosh(\lambda_1 h_1)}d\alpha$$

$$\sigma_{yz}(x, y, s) = -\frac{\mu C_{44}B_z(s)}{2}\frac{\beta_1\beta_2\sinh(\beta_1 h_1)\sinh[\beta_2(h + y)]}{C_{44}\beta_1\cosh(\beta_2 h)\sinh(\beta_1 h_1) + \mu\beta_2\sinh(\beta_2 h)\cosh(\beta_1 h_1)}$$
$$+\frac{\mu C_{44}B_z(s)}{\pi}\int_0^\infty \frac{\lambda_1\lambda_2\sinh(\lambda_1 h_1)\sinh[\lambda_2(h + y)]\sin[\alpha(x - \xi)]}{\alpha[C_{44}\lambda_1\cosh(\lambda_2 h)\sinh(\lambda_1 h_1) + \mu\lambda_2\sinh(\lambda_2 h)\cosh(\lambda_1 h_1)]}d\alpha, \quad -h < y < 0 \tag{14}$$