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Letter

Electromechanical phase transition of a dielectric elastomer tube under internal pressure of constant mass

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ABSTRACT

The electromechanical phase transition for a dielectric elastomer (DE) tube has been demonstrated in recent experiments, where it is found that the unbulged phase gradually changed into bulged phase. Previous theoretical works only studied the transition process under pressure control condition, which is not consistent with the real experimental condition. This paper focuses on more complex features of the electromechanical phase transition under internal pressure of constant mass. We derive the equilibrium equations and the condition for coexistent states for a DE tube under an internal pressure, a voltage through the thickness and an axial force. We find that under mass control condition the voltage needed to maintain the phase transition increases as the process proceeds. We analyze the entire process of electromechanical phase transition and find that the evolution of configurations is also different from that for pressure control condition.

Dielectric elastomer (DE) has drawn extensive attention in last several years because of the outstanding attributes including large deformation, fast response to external stimuli, light weight and low cost [1–3]. These properties make DE promising for massive applications in designing novel soft actuators [4–6], sensors [7–9], and generators [10–12]. When actuated by mechanical force and voltage simultaneously, certain DE structures may deform into a coexistent state with combinations of different homogeneous stretch states, for example, flat part and wrinkled part coexist on a pre-stretched DE membrane under a fixed voltage [13,14], and the bulged part coexists with unbulged part in a circular DE balloon under air pressure and voltage [15,16]. The experimental observations have been explained by the theory of coexistence from thermodynamics [17]. We call the process that a DE structure deforms from a single homogeneous state to a coexistent state under electromechanical coupling loading as electromechanical phase transition.

The electromechanical phase transition of a DE structure is highly similar to the phase transition of water [18]. If a container of water is heated to boil, it changes from liquid state to gaseous state gradually. The phase transition process ends when all of the liquid becomes gas. During the transition process, the water changes into a coexistent state of liquid phase and gaseous phase. In the coexistent state, we keep heating the water and the entropy increases, thus the proportion of the gaseous phase increases and the liquid phase decreases. When the water–vapor system is open to the atmosphere, as the transition process proceeds, the vapor goes into the atmosphere. Since the volume of the atmosphere environment is infinitely larger than the volume of the container, the pressure of the system stays unchanged. Under this pressure control condition, the boiling temperature stays invariant during the phase transition, as shown in the horizontal line in Fig. 1(a). The above-mentioned phenomenon is familiar to us when we boil water in daily life.

When the water and vapor are sealed in a closed container (Fig. 1(b)), upon heating the water, the liquid phase also changes into gaseous phase gradually. Under this condition with internal pressure of constant mass, as the proportion of gaseous phase increases in the closed system, the pressure in the closed container increases according to the ideal gas law. In this case, the boiling point increases correspondingly, resulting in an ascending temperature during the phase transition process (Fig. 1(b)). This phenomenon is also familiar to us when we cook food using a pressure-cooker.

A recent experiment reported that a DE tubular balloon undergoes electromechanical phase transition under the condition with internal pressure of constant mass [19]. In the experimental setup, pressure control is usually difficult while mass control is easier by using a connected air chamber. Previous studies focus

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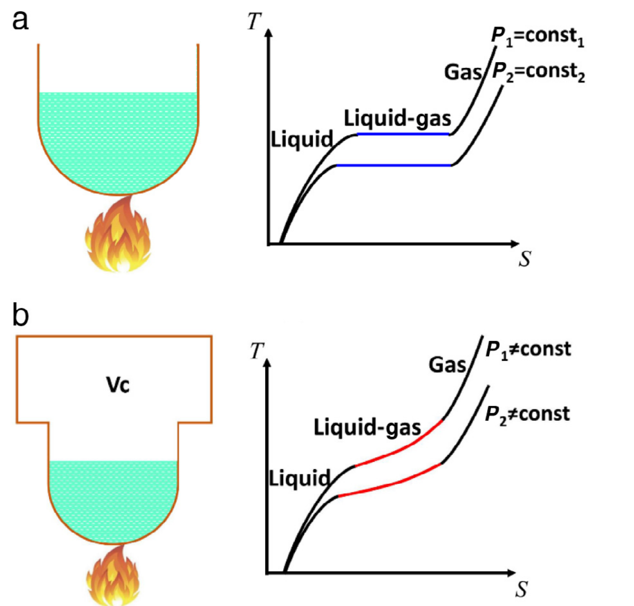

Fig. 1. Phase transition of water (temperature–entropy diagram). (a) When the water is open to atmosphere, the boiling temperature keeps constant during the phase transition. (b) When the water and vapor are sealed in a container, the boiling temperature increases during the phase transition.

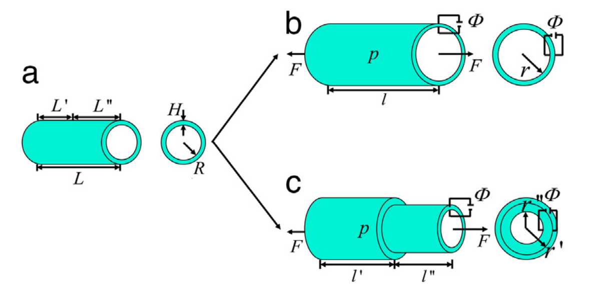

Fig. 2. Schematic of the DE tube. (a) Reference state when the DE tube is subject to no external loads. When subject to an internal pressure p, a voltage Φ and an axial force F, the tube may deform (b) homogeneously or (c) inhomogeneously.

on the electromechanical phase transition of a dielectric elastomer tube under a constant pressure and voltage and no work studies the phase transition under mass control condition, which is the real situation in experiments. Inspired from the phase transition of water under mass control, this work aims to find the common features that may exist for the electromechanical phase transition for a DE tubular balloon.

We refer to the similar procedure described in Ref. [16] to establish the governing equations for the homogeneous deformation of a dielectric elastomer tubular balloon under an internal pressure, a voltage through the thickness and an axial constant force. Herein the internal pressure is not constant any more, rather it is constrained by the number of closed gas molecules of the system and varies as the tube deforms. In the reference state, the balloon is not subject to any external load. The initial tube is of thickness H, radius R and length L, and the volume enclosed is $V_0 = \pi R^2 L$, shown in Fig. 2(a). The balloon is connected to a chamber with size V_c. The balloon together with the chamber seals the gas inside as a closed system. Two compliant electrodes are coated on the surfaces inside and outside of the DE balloon. The tube is deformed by an internal pressure p and a voltage Φ through the thickness. After deformation, the thickness of the tube becomes h, the radius becomes r, the length becomes l, and the charges accumulated on both electrodes are $\pm Q$, shown in Fig. 2(b). We assume the membrane to be incompressible, so that $RHL = rhl$. Define the longitudinal stretch of the tube as $\lambda_1 = l/L$, the hoop stretch as $\lambda_2 = r/R$, so the volume of the tubular balloon is $v = \pi r^2 l = \pi R^2 L \lambda_1 \lambda_2^2$.

The tube and the mechanisms that apply the pressure and voltage form a composite thermodynamic system. Assuming the temperature to be kept constant, this composite exchanges energy with the environment by heat. Under the isothermal condition, when the Helmholtz free energy of the composite reaches a stationary state, the composite reaches equilibrium. The thermodynamic state of tube element is described by the stretches λ_1, λ_2 and electric displacement D. The Helmholtz free energy of the tube can be represented as $2\pi RHL\left[W_s(\lambda_1,\lambda_2) + D^2/(2\varepsilon)\right]$, which includes the elastic energy and the electrostatic energy [20]. The potential energy of the mechanism that applies the pressure can be represented as $-pv$, that applies the voltage $-\Phi Q$, and that applies the constant force $-Fl$. Summing up, the total Helmholtz free energy of this thermodynamic system is

$$\Pi(\lambda_1,\lambda_2,D) = 2\pi RHL\left[W_s(\lambda_1,\lambda_2) + D^2/(2\varepsilon)\right] - \Phi Q - Fl - pv. \tag{1}$$

Taking the stiffening effect of polymers into consideration, we adopt the Gent material model [21] with the form

$$W_s(\lambda_1,\lambda_2) = -\frac{\mu}{2} J_{\lim} \log\left(1 - \frac{\lambda_1^2 + \lambda_2^2 + \lambda_1^{-2}\lambda_2^{-2} - 3}{J_{\lim}}\right). \tag{2}$$

The electric field is $E = \Phi/h = \lambda_1\lambda_2\Phi/H$, and the charge on each electrode is $Q = 2\pi rlD$. Setting the partial derivatives with respect to the three independent variables to zero, we obtain the equilibrium equations,

$$D = \varepsilon\lambda_1\lambda_2\phi/H, \tag{3}$$

$$\lambda_1\lambda_2^2\frac{pR}{2H} + \lambda_1\frac{F}{2\pi RH} + \varepsilon(\lambda_1\lambda_2)^2\left(\frac{\phi}{H}\right)^2 = \lambda_1\frac{\partial W_s(\lambda_1,\lambda_2)}{\partial\lambda_1}, \tag{4}$$

$$\lambda_1\lambda_2^2\frac{pR}{H} + \varepsilon(\lambda_1\lambda_2)^2\left(\frac{\phi}{H}\right)^2 = \lambda_2\frac{\partial W_s(\lambda_1,\lambda_2)}{\partial\lambda_2}. \tag{5}$$

Equation (3) recovers the ideal dielectric elastomer model [20]. Once the loading parameters p and Φ are given, Eqs. (4) and (5) will determine the stretches λ_1 and λ_2. In the initial state, the chamber seals a part of gas with initial pressure p_0 and is unconnected with the tube. In the actuated state, the tube deforms and is connected with the chamber, the common pressure becomes p. According to the ideal gas law we have the constraint for the pressure and volume as

$$(\pi R^2 L\lambda_1\lambda_2^2 + V_c)(p + p_a) = \pi R^2 L p_a + V_c(p_0 + p_a), \tag{6}$$

where p_a is the atmosphere pressure. We use dimensionless volume $\frac{V_c}{\pi R^2 L}$ to measure the size of the chamber relative to the tube. The applied voltage deforms the tube, described by λ_1 and λ_2, and then the deformation leads to another balanced pressure according to the ideal gas law. The pressure p is specified by the applied voltage and the current deformation state.

Subject to the applied pressure and voltage, the tube may also deform inhomogeneously (Fig. 2(c)), where a bulged and an unbulged section equilibrate [16]. In the reference state, the bulged and unbulged sections are of lengths L' and L''. When the tube is actuated, the bulged and unbulged sections are of lengths l' and l'', volumes v' and v'', charges Q' and Q'', electric displacements D' and D'', and stretches (λ_1', λ_2') and $(\lambda_1'', \lambda_2'')$, shown in Fig. 2(c). The Helmholtz free energy of the balloon consists of both the elastic energy and the dielectric energy: $2\pi RHL'[W_s(\lambda_1',\lambda_2') + D'^2/(2\varepsilon)] + 2\pi RHL''[W_s(\lambda_1'',\lambda_2'') + D''^2/(2\varepsilon)]$. The potential energy of the mechanism that applies the voltage and the pressure can be represented as $-\Phi(Q' + Q'')$ and $-p(v' + v'')$, respectively. The