ARTICLE IN PRESS

Theoretical & Applied Mechanics Letters ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Theoretical & Applied Mechanics Letters

journal homepage: www.elsevier.com/locate/taml

Letter

Effect of negative permeability on elastic wave propagation in magnetoelastic multilayered composites

W. Wang [a], Y.Q. Guo [a,*], W.Q. Chen [b,c]

[a] Key Laboratory of Mechanics on Disaster and Environment in Western China, Ministry of Education, and School of Civil Engineering and Mechanics, Lanzhou University, Lanzhou 730000, China

[b] Department of Engineering Mechanics, Yuquan Campus, Zhejiang University, Hangzhou 310027, China

[c] Key Laboratory of Soft Machines and Smart Devices of Zhejiang Province, Zhejiang University, Hangzhou 310027, China

HIGHLIGHTS

- The method of reverberation-ray matrix is extended for the analysis.
- Negative besides positive permeability of the piezomagnetic layers is considered.
- Dispersion curves of elastic waves in magnetoelastic multilayers are computed.
- Novel properties of elastic wave propagation like mode conversion are found.

ARTICLE INFO

Article history:
Received 9 December 2016
Accepted 19 January 2017
Available online xxxx
*This article belongs to the Solid Mechanics

Keywords:
Magnetoelastic multilayered composites
Elastic wave propagation
Method of reverberation-ray matrix
Negative permeability
Wave mode conversion

ABSTRACT

With the advent of left-handed magnetic materials, it is desirable to develop high-performance wave devices based on their novel properties of wave propagation. This letter reports the special properties of elastic wave propagation in magnetoelastic multilayered composites with negative permeability as compared to those in counterpart structures with positive permeability. These novel properties of elastic waves are discerned from the diversified dispersion curves, which represent the propagation and attenuation characteristics of elastic waves. To compute these dispersion curves, the method of reverberation-ray matrix is extended for the analysis of elastic waves in magnetoelastic multilayered composites. Although only the results of a single piezomagnetic and a binary magnetoelastic layers with mechanically free and magnetically short surfaces as well as perfect interface are illustrated in the numerical examples, the analysis is applicable to magnetoelastic multilayered structures with other kinds of boundaries/interfaces.

Magnetoelastic multilayered composites consist of stacked piezomagnetic and elastic layers. Due to the magnetic–mechanical coupling in piezomagnetic materials and the bonding between piezomagnetic and elastic layers, elastic wave propagation in these composites is related to the magnetic field. Based on this feature, magnetoelastic multilayered composites can be used for developing wave devices involving mechanical and magnetic fields [1], which also has stimulated many academic interests [2–7]. However, the magnetic permeability adopted in these investigations [2–7] is positive without exception, possibly because almost all piezomagnetic materials in nature have positive permeability.

The negative magnetic permeability and electric permittivity was first theoretically hypothesized in 1968 by Veselago [8], who predicted that negative refraction of electromagnetic waves could occur in substances of this kind called as left-handed material. This originally hypothetical double negative material has come true at the beginning of this century, which is owed to Smith et al. [9]. The novel properties of electromagnetic waves in this metamaterial such as negative refraction [10,11] and reversed Cherenkov radiation [12] have also been confirmed, which may lead to various applications [13,14]. Recently, analogy to double negative electromagnetic metamaterial for controlling electromagnetic wave, the double negative acoustic

* Corresponding author.
E-mail address: guoyq@lzu.edu.cn (Y.Q. Guo).

http://dx.doi.org/10.1016/j.taml.2017.01.006

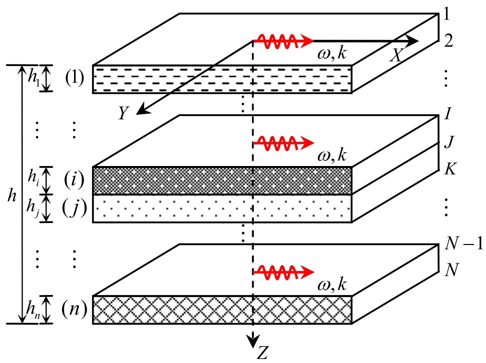

Fig. 1. Schematic and description of the magnetoelastic composite in its global coordinate system.

metamaterial for controlling elastic (acoustic) wave has been presented and investigated [15,16].

According to these concepts, we propose to construct magnetoelastic multilayered composites by employing piezomagnetic layers with negative permeability [17]. Because of magnetic–mechanical coupling in magnetoelastic multilayered composites, novel properties of elastic waves are possible on account of the negative magnetic permeability, based on which high-performance wave devices may be developed. In this letter, by comparing the dispersion curves as all magnetic permeabilities are positive with the dispersion curves as some or all permeabilities reverse their sign, we search for novel properties of elastic waves in transversely isotropic magnetoelastic multilayered composites with negative permeabilities. To analyze these dispersion curves, the method of reverberation-ray matrix (MRRM) [18,19] is introduced to derive the dispersion equation that is solved numerically through a root searching method [19].

The transversely isotropic magnetoelastic multilayered composite consists of n perfectly stacked and unbounded layers of piezomagnetic/elastic materials, whose schematic together with the global coordinate system $OXYZ$ is given in Fig. 1. The involved piezomagnetic and elastic materials are assumed to be of magnetic crystal classes [20] of (6$\underline{2}$2, 6$\underline{m}$ $\underline{m}$, $\bar{6}\underline{m}$ $\underline{2}$ or $6/m\underline{m}$ $\underline{m}$) and ($6/\underline{m}$, $\underline{6}/m$, $6/\underline{m}$ $\underline{m}$ $\underline{m}$, $6/\underline{m}mm$ or $\underline{6}/mm\underline{m}$), respectively, all with crystallographic axes along the OZ direction. Thus, the property of elastic wave in any direction on the XOY plane is identical and we assume the elastic wave is along the X-axis without loss of generality. The considered elastic wave has circular frequency ω and wavenumber k, as also depicted in Fig. 1. From top to bottom in sequence, the constituent layers are labeled by numbers $(1), (2), \ldots, (n)$, while the surfaces and interfaces are denoted as $1, 2, \ldots, N$ $(N = n+1)$. The thickness of any layer (j) $[(j) = (1), (2), \ldots, (n)]$ is h_j, which contributes to the total thickness h of the whole structure.

According to the MRRM [18,19], pairs of dual local coordinate systems are established for all constituent layers, as shown in Fig. 2. The physical variables of any layer, (j) for instance, will be described in its dual local coordinate systems, $(x, y, z)^{JK}$ and $(x, y, z)^{KJ}$, and the superscripts JK or KJ are attached to represent the pertaining coordinate system. In Fig. 2, u^{JK}, v^{JK}, and w^{JK} are the displacements along the x^{JK}, y^{JK}, and z^{JK} axes, respectively, while τ_{zx}^{JK}, τ_{zy}^{JK}, and σ_z^{JK} are the corresponding stresses on the z^{JK} plane. ψ^{JK} and B_z^{JK} are the magnetic scalar potential and the magnetic flux density along the z^{JK} axis, respectively. $\boldsymbol{v}_u^{JK} = \left[u^{JK}, v^{JK}, w^{JK}, \psi^{JK}\right]^{\mathrm{T}}$ and $\boldsymbol{v}_\sigma^{JK} = \left[\tau_{zx}^{JK}, \tau_{zy}^{JK}, \sigma_z^{JK}, B_z^{JK}\right]^{\mathrm{T}}$ are called as the generalized displacement vector and the generalized stress vector at any point on the z^{JK} plane, which are combined to form the state vector $\boldsymbol{v}^{JK} = \left[(\boldsymbol{v}_u^{JK})^{\mathrm{T}}, (\boldsymbol{v}_\sigma^{JK})^{\mathrm{T}}\right]^{\mathrm{T}}$. The physical variables and state vectors of layer (j) in $(x, y, z)^{KJ}$ coordinates and of other layers in their dual local coordinates are described in a similar way.

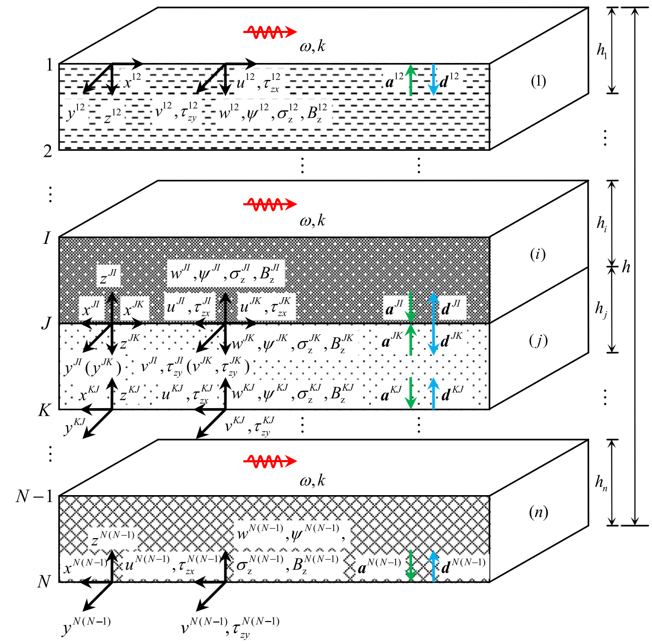

Fig. 2. Description of the constituent layers in their dual local coordinate systems: the generalized displacements and stresses and the wave vectors.

According to the three-dimensional linear theories of elasticity and magnetostatics of transversely isotropic piezomagnetic and elastic materials [2,21,22], the dynamic equations governing the state vector of a constituent layer in any one of its dual local coordinate systems can be derived. In the following, we will omit the superscripts denoting the coordinates for brevity, unless otherwise stated. First, the equations of motion (without body force) and the Gauss's law for magnetism of magnetostatics are expressed as

$$\boldsymbol{L}\boldsymbol{\sigma} = \rho\frac{\partial^2 \boldsymbol{u}}{\partial t^2}, \quad \nabla \cdot \boldsymbol{B} = 0, \tag{1}$$

where $\boldsymbol{u} = [u, v, w]^{\mathrm{T}}$, $\boldsymbol{\sigma} = [\sigma_x, \sigma_y, \sigma_z, \tau_{yz}, \tau_{zx}, \tau_{xy}]^{\mathrm{T}}$ and $\boldsymbol{B} = [B_x, B_y, B_z]^{\mathrm{T}}$ are the displacement, stress and magnetic flux density vectors, respectively. ρ is the material density. $\boldsymbol{L}$ and ∇ are differential operator matrices of 3×6 and of 3×1, respectively:

$$\boldsymbol{L} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 & 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \\ 0 & 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \end{bmatrix}, \quad \nabla = \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix}. \tag{2}$$

Second, the equation between the strain vector $\boldsymbol{\varepsilon} = [\varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{yz}, \gamma_{zx}, \gamma_{xy}]^{\mathrm{T}}$ and the displacement vector $\boldsymbol{u} = [u, v, w]^{\mathrm{T}}$ as well as the relation between the magnetic field intensity vector $\boldsymbol{H} = [H_x, H_y, H_z]^{\mathrm{T}}$ and the magnetic scalar potential ψ are

$$\boldsymbol{\varepsilon} = \boldsymbol{L}^{\mathrm{T}}\boldsymbol{u}, \quad \boldsymbol{H} = -\nabla\psi. \tag{3}$$

The constitutive relations are dependent on the material type. For transversely isotropic piezomagnetic materials (magnetic crystal classes 6$\underline{2}$2, 6$\underline{m}$ $\underline{m}$, $\bar{6}\underline{m}$ $\underline{2}$ or $6/m\underline{m}$ $\underline{m}$) polarized along z-axis [20], the constitutive relations read

$$\boldsymbol{\sigma} = \boldsymbol{c}\boldsymbol{\varepsilon} - \boldsymbol{q}^{\mathrm{T}}\boldsymbol{H}, \quad \boldsymbol{B} = \boldsymbol{q}\boldsymbol{\varepsilon} + \boldsymbol{\mu}\boldsymbol{H}, \tag{4}$$