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Letter

Bending of functionally graded nanobeams incorporating surface effects based on Timoshenko beam model

Lihong Yang [a,*], Tao Fan [a], Liping Yang [b], Xiao Han [a], Zongbing Chen [a]

[a] College of Aerospace and Civil Engineering, Harbin Engineering University, Harbin 150001, China

[b] Engineering Training Centre, Harbin Engineering University, Harbin 150001, China

HIGHLIGHTS

- Analyzing the bending of functionally graded nanobeams based on Timoshenko beam theory.
- Considering surface stress effects of nanobeams by adopting the Gurtin–Murdoch theories.
- Deriving the governing equations by using the principle of minimum total potential energy.
- Investigating the influences of gradient index and surface stress on the bending responses.

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ABSTRACT

The bending responses of functionally graded (FG) nanobeams with simply supported edges are investigated based on Timoshenko beam theory in this article. The Gurtin–Murdoch surface elasticity theory is adopted to analyze the influences of surface stress on bending response of FG nanobeam. The material properties are assumed to vary along the thickness of FG nanobeam in power law. The bending governing equations are derived by using the minimum total potential energy principle and explicit formulas are derived for rotation angle and deflection of nanobeams with surface effects. Illustrative examples are implemented to give the bending deformation of FG nanobeam. The influences of the aspect ratio, gradient index, and surface stress on dimensionless deflection are discussed in detail.

Nanoscale structures are widely used in various engineering fields due to the specific physical and mechanical properties. Surface effects have an important influence on mechanical properties of nanoscale structures owing to large surface-to-bulk ratio. Poncharal et al. [1] studied the bending modulus of carton nanotubes by experiment methods and found it increased dramatically with decreasing diameters. Some researchers focused on the size-dependence of nanostructures by experiment or theoretical methods [2–6]. The approaches of studying mechanical behavior of nanoscale structures include experiments method, atomistic simulation method, and continuum mechanics method, etc. The application of continuum mechanics method becomes more and more extensive because performing the controlled nanoscale experiments is very difficult and computational capacity of computers limits the atomistic simulation methods [7]. Gurtin and Murdoch [8,9] developed a linear elastic surface effects theory based on continuum mechanics in which the surface of nanostructure is regarded as a membrane of zero thickness, and this membrane is assumed to be fully adhered to the bulk material. Gurtin–Murdoch surface elasticity theory has an important effect on the development of continuum mechanics method. Recently, researchers in various countries implemented a large number of investigations on the influence of surface effects on mechanical properties of nanostructures by using Gurtin–Murdoch surface elasticity theory [10–13]. To give a more accurate analysis of nanostructure, a continuum model of surface elasticity was formulated by extending Laplace–Young equation which was established to address the surface/interface tension of fluids to solid materials by Gurtin et al. [14]. Then, the generalized Laplace–Young equation of curved surface in nanostructures was derived by Chen et al. [15].

As a new kind of composite materials, functionally graded materials (FGMs) which have the continuous variation of material properties have been applied in different fields of science and

* Corresponding author.
E-mail address: yanglihong1@hrbeu.edu.cn (L. Yang).

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technology (for instance, optoelectronics, nanotechnology, tribology and high temperature technology, etc.). FGMs are generally composed of two constituents and volume fraction of each constituent varies continuously across the functionally graded (FG) body. Many researchers implemented bending, buckling, and vibration analysis of FG beams [16–18].

In recent years, FG structures had been applied in micro/nanoelectro mechanical systems [19–26], and the mechanical analysis of FG nanostructures has become one of the attractive research hotspots. Differential quadrature method was used to investigate free vibration in axially functionally graded carbon nanotubes based on the Timoshenko beam theory (TBT) [27]. FG nanobeam [28–33] and FG nanoplate [34–36] were analyzed extensively on bending, buckling, and vibration. Euler–Bernoulli nanobeams made of bi-directional functionally graded materials were investigated by Mohammad et al. on buckling [37], bending [38], and free vibration [39] based on Eringen's non-local elasticity theory.

According to the above discussion, understanding the influence of the surface effects on mechanical behavior of FG nanobeams has an important role in the design of nanodevices. Timoshenko beam theory considering the influence of transverse shear strain is more appropriate to analyze mechanical behavior of the moderately deep beam than the conventional Euler–Bernoulli beam theory (EBT). In the present research, we focus on the bending of FG nanobeams on the basis of Gurtin–Murdoch surface elasticity theory and Timoshenko beam theory. The material properties of FG nanobeam are assumed to vary in power law along the thickness of beam. The principal of the minimum total potential energy is adopted to determine the governing equations and the corresponding boundary conditions. The exact solution for the bending deflections is proposed under simply-supported boundary conditions. The effects of aspect ratio, gradient index and surface elastic parameters on the deflection of the FG nanobeam are discussed. The obtained solutions are also verified by conducting some illustrative examples.

Figure 1 shows an FG nanobeam of length L, thickness h, and width b. Both distributed transverse pressure $q(x)$ and an axial compressive force P act on this nanobeam. FG nanobeam is assumed to be made of two different constituents and the effective material properties are assumed to vary continuously along the thickness of beam. According to the rule of mixture, the effective material properties P (i.e., bulk elastic modulus E, surface elastic modulus E^s, and residual surface stress τ_0) can be expressed as

$$P = P_1V_1 + P_2V_2, \tag{1}$$

where the subscripts 1 and 2 represent the first constituents and the second constituents, respectively. $P_i\ (i = 1, 2)$ is the effective material properties of constituent materials, and $V_i\ (i = 1, 2)$ is the corresponding volume fractions which are assumed to change in power law in z direction

$$V_1(z) = 1 - \left(\frac{z}{h} + \frac{1}{2}\right)^k, \tag{2}$$

$$V_2(z) = \left(\frac{z}{h} + \frac{1}{2}\right)^k, \tag{3}$$

where k is the gradient index (or the volume fraction index).

Then bulk elastic modulus $E(z)$, surface elastic modulus $E^s(z)$, and residual surface stress $\tau_0(z)$ of the FG nanobeam can be derived respectively in the following form

$$E(z) = (E_2 - E_1)\left(\frac{z}{h} + \frac{1}{2}\right)^k + E_1, \tag{4}$$

$$E^s(z) = \left(E_2^s - E_1^s\right)\left(\frac{z}{h} + \frac{1}{2}\right)^k + E_1^s, \tag{5}$$

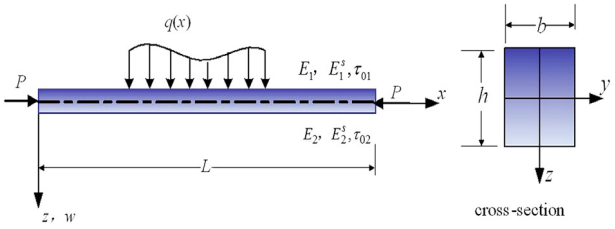

Fig. 1. Schematic of FG nanobeam.

$$\tau_0(z) = (\tau_{02} - \tau_{01})\left(\frac{z}{h} + \frac{1}{2}\right)^k + \tau_{01}, \tag{6}$$

where τ_{01} and τ_{02} are residual surface stresses of two constituents. Poisson's ratio ν of FG nanobeam is generally taken as constant.

Due to no slipping between upper (below) surface and the bulk material of FG nanobeam, there exist the continuous displacements in the whole nanobeam. Based on Timoshenko beam theory, the axial displacement u_1 and the transverse displacement u_3 at arbitrary point (x, z) of the nanobeam can be given as

$$u_1 = u - z\varphi, \tag{7}$$

$$u_3 = w, \tag{8}$$

where u and w are axial displacements and transverse displacements for arbitrary point $(x, 0)$ on the neutral axis, respectively, and φ is the rotation angle of cross-section with respect to the neutral axis. The strain–displacement relationship of Timoshenko beam theory can be expressed as

$$\varepsilon_{xx} = \frac{\mathrm{d}u}{\mathrm{d}x} - z\frac{\mathrm{d}\varphi}{\mathrm{d}x}, \tag{9}$$

$$\gamma_{xz} = \frac{\mathrm{d}w}{\mathrm{d}x} - \varphi. \tag{10}$$

Assuming residual stress in the bulk is negligible due to surface energy, the bulk constitutive equations of FG nanobeam can be given by

$$\sigma_{xx} = E(z)\left(\frac{\mathrm{d}u}{\mathrm{d}x} - z\frac{\mathrm{d}\varphi}{\mathrm{d}x}\right), \tag{11}$$

$$\sigma_{xz} = \psi G(z)\left(\frac{\mathrm{d}w}{\mathrm{d}x} - \varphi\right), \tag{12}$$

in which ψ is the shear correction factor and equals 5/6 for a rectangular cross section. Shear elastic modulus $G(z) = E(z)/(2 + 2\nu)$.

To analyze the surface stress effects of nanostructure, a theoretical model is developed by Gurtin and Murdoch [8,9] based on the elasticity continuum mechanics including surface stress effects. And the following surface constitutive equations were proposed [8,9]

$$\sigma_{\alpha\beta}^s = \tau_0\delta_{\alpha\beta} + \left(\tau_0 + \lambda^s\right)\varepsilon_{\gamma\gamma}\delta_{\alpha\beta} + 2\left(\mu^s - \tau_0\right)\varepsilon_{\alpha\beta} + \tau_0 u_{\alpha,\beta}^s, \tag{13}$$

$$\sigma_{\alpha z}^s = \tau_0 u_{z,\alpha}^s, \tag{14}$$

where $\alpha, \beta = x, y$, the superscript s is applied to denote the quantities corresponding to the surface layer, and λ^s, μ^s are the Lame constants of the surface.

The surface constitutive equations for FG nanobeam can be obtained from Eqs. (13) and (14) as

$$\sigma_{xx}^s = E^s(z)\,\varepsilon_{xx} + \tau_0(z) = E^s(z)\left(\frac{\mathrm{d}u}{\mathrm{d}x} - z\frac{\mathrm{d}\varphi}{\mathrm{d}x}\right) + \tau_0(z), \tag{15}$$

$$\sigma_{xz}^s = \tau_0(z)\frac{\mathrm{d}w}{\mathrm{d}x}, \tag{16}$$

where the surface elastic modulus $E^s = 2\lambda^s + \mu^s$.