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Article

Buckling and post-buckling analyses of size-dependent piezoelectric nanoplates

Chen Liu [a,b], Liao-Liang Ke [a,*], Jie Yang [b], Sritawat Kitipornchai [c], Yue-Sheng Wang [a]

[a] Institute of Engineering Mechanics, Beijing Jiaotong University, Beijing 100044, China

[b] School of Aerospace, Mechanical and Manufacturing Engineering, RMIT University, PO Box 71, Bundoora, VIC 3083, Australia

[c] School of Civil Engineering, The University of Queensland, St Lucia, QLD 4072, Australia

HIGHLIGHTS

- An increasing nonlocal parameter leads to smaller buckling and post-buckling loads.
- The positive/negative electric voltage decreases/increases buckling and post-buckling loads.
- A relatively large temperature rise results in slight drops in buckling and post-buckling loads.

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ABSTRACT

This paper attempts to investigate the buckling and post-buckling behaviors of piezoelectric nanoplate based on the nonlocal Mindlin plate model and von Karman geometric nonlinearity. An external electric voltage and a uniform temperature rise are applied on the piezoelectric nanoplate. Both the uniaxial and biaxial mechanical compression forces will be considered in the buckling and post-buckling analysis. By substituting the energy functions into the equation of the minimum total potential energy principle, the governing equations are derived directly, and then discretized through the differential quadrature (DQ) method. The buckling and post-buckling responses of piezoelectric nanoplates are calculated by employing a direct iterative method under different boundary conditions. The numerical results are presented to show the influences of different factors including the nonlocal parameter, electric voltage, and temperature rise on the buckling and post-buckling responses.

1. Introduction

Beam or plate type piezoelectric nanostructures have been extensively applied in many branches of nano-electro-mechanical systems (NEMS, e.g. nanoresonator, nanogenerator, light-emitting diodes, chemical sensors, etc.), thus researches on the piezoelectric nanostructures have been active research fields with worldwide attention [1–3]. In practical applications, it is common for the beam or plate type components of nanodevices to suffer in-plane compressive loadings, which may cause buckle failure of the nanostructures, thus studies on the buckling and post-buckling behaviors of the piezoelectric nanostructures are important in the design of nanodevices.

Compared with their macroscale counterparts, piezoelectric nanostructures are found of possessing superior thermal, electrical, mechanical, and other physical/chemical properties [4,5]. As generally believed, the size-dependent properties become significant as the dimensions of the nanostructures vary from several hundred nanometers to just a few nanometers. Both experiments and atomic simulations [6,7] show valid evidences of the fact that the long-range inter atomic and inter molecular cohesive forces gain significant and non-ignorable influences on the mechanical properties when the length scale reduces. It is well-known that the nonlocal theory raised by Eringen [8–10] is considered as an effective theory to characterize the size effect of nanostructures. Unlike the classical size-independent continuum theory, the nonlocal theory introduces the internal length scale into the constitutive equations to account the scale effect in micro- and nano-scaled structures. The key point in the nonlocal theory is that, at a reference point, the stress depends on not only the strain component at the point, but also all the strain components in the domain around. Such proposal is widely accepted to study the size-dependent properties of nanostructures, including the bending [11–14], buckling and post-buckling [15–18], linear and nonlinear vibrations [19–24], and wave propagation characteristics [25,26] of carbon nanotubes, graphene sheets, nanowires, etc.

* Corresponding author. Fax: +86 10 51682094.
E-mail address: llke@bjtu.edu.cn (L.-L. Ke).

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Besides the analysis of elastic nanostructures mentioned above, applications of the nonlocal theory already have been extended to the size-dependent electro-mechanical analysis of the piezoelectric nanostructures in recent years. Ke and his co-authors have examined the free vibration behavior of nonlocal piezoelectric nanoplates by deriving analytical solutions based on the Kirchhoff plate theory [27] and numerical solutions based on the Mindlin plate theory [28]. The nonlocal plate model is also employed by Asemi and his co-authors to study the vibration characteristics of piezoelectric nanoplate systems embedded in Pasternak elastic medium [29] and of double piezoelectric nanoplate systems with initial stress [30]. Arani and his co-authors [31–34] presented comprehensive investigations on the boron nitride nanotubes (BNNTs) by using nonlocal nanobeam and nanoshell models, to study the buckling, linear, and nonlinear vibrations under electro-mechanical environments. Wang and Wang [35] examined the bending behavior of a piezoelectric nanowire. Their study included both the surface effect and size effect by using the surface elasticity theory and nonlocal theory. Zhang et al. [36] also considered both nonlocal effect and surface effect in the analysis of the dispersion characteristics of the elastic wave propagating in a monolayer piezoelectric nanoplate.

For the piezoelectric nanoplate subjected to in-plane loadings on its edges, the compression forces may result in nonlinear deformation at high loading levels. To accurately predict the post-buckling response in such cases, the geometrical nonlinearity should be considered as an essential influencing factor on the post-buckling performance. Some investigations have been already presented in the field of nonlinear analysis on the piezoelectric nanostructures. Based on the von Karman type nonlinear displacement–strain relationship, Ke and his co-authors investigated the nonlinear vibration [37] and post-buckling [38] behaviors of nonlocal Timoshenko piezoelectric nanobeams under combined thermo-electro-mechanical loadings. The von Karman nonlinear equations are also applied by Asemi et al. [39] to establish an analytical procedure to investigate nonlinear vibration and post-buckling behaviors of piezoelectric nano-electro-mechanical resonators based on the simply supported Kirchhoff piezoelectric nanofilm (PNF) model. More recently, based on the nonlocal Kirchhoff plate theory, Liu et al. [40] presented the analytical study on the nonlinear vibration of the simply supported piezoelectric nanoplates rested on Winkler foundations.

This paper attempts to investigate the buckling and post-buckling behaviors of the piezoelectric nanoplates subjected to combined thermo-electro-mechanical loadings based on the nonlocal theory, Mindlin plate theory and von Karman geometric nonlinearity. By using the principle of minimum total potential energy, the nonlinear governing equations and corresponding boundary conditions can be derived. The equations are then discretized by using the differential quadrature (DQ) method and solved by a direct iterative method to determine the buckling and post-buckling responses of piezoelectric nanoplates with different boundary conditions. Parametric studies are presented to show the influences of the nonlocal parameter, electric voltage and temperature rise on the size-dependent buckling and post-buckling responses.

2. Nonlocal theory for piezoelectric materials

The traditional local elasticity is size independent, thus it fails to predict the mechanical properties of nanostructures. The nonlocal theory raised by Eringen [8–10] overcomes the shortage by introducing a simple physical concept that the component of stress tensor at a certain point is a function not only of strain tensor at the point but also of strain tensors in the domain. Recently, Ke and his co-authors [27,28] presented some works applying the nonlocal theory in the analysis of piezoelectric nanostructures. According to Ke et al. [27,28], the nonlocal constitutive relations for the piezoelectric solids may be expressed in differential forms as

$$\sigma_{ij} - (e_0 a)^2 \nabla^2 \sigma_{ij} = c_{ijkl}\varepsilon_{kl} - e_{kij}E_k - \lambda_{ij}\Delta T, \tag{1}$$

$$D_i - (e_0 a)^2 \nabla^2 D_i = e_{ikl}\varepsilon_{kl} - \kappa_{kij}E_k + p_i\Delta T, \tag{2}$$

where σ_{ij}, ε_{ij}, D_i, and E_i are the components of stress, strain, electric displacement, and electric field, respectively; c_{ijkl}, e_{kij}, κ_{kij}, λ_{ij}, and p_i are the elastic constant, piezoelectric constant, dielectric constant, thermal modulus, and pyroelectric constant, respectively; ΔT is the temperature rise; ∇^2 is the Laplacian operator; $e_0 a$ is the scale coefficient incorporating the size effect on the response of structures in nanosize where a is an internal characteristic length (e.g., lattice parameter, granular size). The parameter e_0 is essential for the validity of nonlocal models, which can be determined by matching the dispersion curves based on the atomic curves. Specifically, for the piezoelectric nanoplates, the nonlocal relations can be approximated as

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xz}\\ \sigma_{yz}\\ \sigma_{xy}\end{Bmatrix} - (e_0 a)^2 \nabla^2 \begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xz}\\ \sigma_{yz}\\ \sigma_{xy}\end{Bmatrix} = \begin{bmatrix}\tilde{c}_{11} & \tilde{c}_{12} & 0 & 0 & 0\\ \tilde{c}_{12} & \tilde{c}_{11} & 0 & 0 & 0\\ 0 & 0 & \tilde{c}_{44} & 0 & 0\\ 0 & 0 & 0 & \tilde{c}_{44} & 0\\ 0 & 0 & 0 & 0 & \tilde{c}_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ 2\varepsilon_{xz}\\ 2\varepsilon_{yz}\\ 2\varepsilon_{xy}\end{Bmatrix} - \begin{bmatrix}0 & 0 & \tilde{e}_{31}\\ 0 & 0 & \tilde{e}_{31}\\ \tilde{e}_{15} & 0 & 0\\ 0 & \tilde{e}_{15} & 0\\ 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}E_x\\ E_y\\ E_z\end{Bmatrix} - \begin{Bmatrix}\tilde{\lambda}_{11}\\ \tilde{\lambda}_{11}\\ 0\\ 0\\ 0\end{Bmatrix}\Delta T, \tag{3}$$

$$\begin{Bmatrix}D_x\\ D_y\\ D_z\end{Bmatrix} - (e_0 a)^2 \nabla^2 \begin{Bmatrix}D_x\\ D_y\\ D_z\end{Bmatrix} = \begin{bmatrix}0 & 0 & \tilde{e}_{15} & 0 & 0\\ 0 & 0 & 0 & \tilde{e}_{15} & 0\\ \tilde{e}_{31} & \tilde{e}_{31} & 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ 2\varepsilon_{xz}\\ 2\varepsilon_{yz}\\ 2\varepsilon_{xy}\end{Bmatrix} + \begin{bmatrix}\tilde{\kappa}_{11} & 0 & 0\\ 0 & \tilde{\kappa}_{11} & 0\\ 0 & 0 & \tilde{\kappa}_{33}\end{bmatrix}\begin{Bmatrix}E_x\\ E_y\\ E_z\end{Bmatrix} + \begin{Bmatrix}\tilde{p}_1\\ \tilde{p}_1\\ \tilde{p}_3\end{Bmatrix}\Delta T, \tag{4}$$

where $\tilde{c}_{ij}$, $\tilde{e}_{ij}$, $\tilde{\kappa}_{ij}$, $\tilde{\lambda}_{ij}$, and $\tilde{p}_i$ are respectively the reduced elastic constants, piezoelectric constants, dielectric constants, thermal moduli, and pyroelectric constants for the piezoelectric nanoplate under the plane stress state [41,42]. These constants are given as

$$\tilde{c}_{11} = c_{11} - \frac{c_{13}^2}{c_{33}}, \quad \tilde{c}_{12} = c_{12} - \frac{c_{13}^2}{c_{33}}, \quad \tilde{c}_{44} = c_{44}, \quad \tilde{c}_{66} = c_{66}, \quad \tilde{e}_{31} = e_{31} - \frac{c_{13}e_{33}}{c_{33}}, \quad \tilde{e}_{15} = e_{15}, \tag{5a}$$

$$\tilde{\kappa}_{11} = \kappa_{11}, \quad \tilde{\kappa}_{33} = \kappa_{33} + \frac{e_{33}^2}{c_{33}}, \quad \tilde{\lambda}_{11} = \lambda_{11} - \frac{c_{13}\lambda_{33}}{c_{33}}, \quad \tilde{p}_1 = p_1, \quad \tilde{p}_3 = p_3 + \frac{e_{33}\lambda_{33}}{c_{33}}. \tag{5b}$$

3. Buckling and post-buckling of piezoelectric nanoplates

As shown in Fig. 1, a nonlocal continuum model is developed for the buckling and post-buckling analyses of the piezoelec-