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Letter

On the ratio of expectation crossings of random-excited dielectric elastomer balloon

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HIGHLIGHTS

- Stationary response of random-excited dielectric elastomer balloon is derived by combining a specific transformation and the stochastic averaging technique.
- Ratio of expectation crossings of dielectric elastomer balloon is derived from the joint probability density of stretch ratio and its ratio of change.

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ABSTRACT

The ratio of expectation crossings of dielectric elastomer balloon excited by random pressure is analytically evaluated in this letter. The Mooney–Rivlin model is adopted to describe the constitutive relation while the random pressure is described by Gaussian white noise. Through a specific transformation, the stochastic differential equations for the total energy and phase are derived. With the application of the stochastic averaging, the system total energy is then approximated by a one-dimensional diffusion process. Solving the associated Fokker–Planck–Kolmogorov (FPK) equation yields the stationary probability density of the system total energy. The ratio of expectation crossings is then derived based on the joint stationary probability density of stretch ratio and its ratio of change. The efficacy and accuracy of the proposed procedure are verified by comparing with the results from Monte Carlo simulation (MCS).

Recently, dielectric elastomer structures have been extensively investigated in academic communities and adopted as functional components in industrial circles. The electric field imposed on the compliant electrodes which cover the surfaces of a polymer film induces significant shrink in thickness and at the same time expansion in plane. The pre-determined change of electric field yields specific change of expansion. Dielectric elastomer structures have many prominent advantages, such as large stretch, short response time, light weight, low cost, chemical and biological compatibility [1–4], and have been directly applied to many practical devices, such as artificial muscles, adaptive optical elements, soft robots, resonators, and electromechanical transducers [5–8].

As dielectric elastomer structures are adopted as flexible actuator, sensor, and loudspeaker, the performance of devices depends on the dynamical properties of dielectric elastomer structures imposed by electrical and/or mechanical excitations. The dynamical behaviors, such as dynamical response, fundamental natural frequency, bifurcation and resonance, have been investigated as the dielectric elastomer structures excited by time-varying voltage or mechanical force [9–13]. The balloon is one of the important configurations of dielectric elastomer structures and can be used to design pumps and loudspeakers. The researches on the dynamical properties of the dielectric elastomer structures are extremely important. Mockensturm and Goulbourne [14] studied the dynamical response of an inflated spherical membrane to an external voltage, and the occurrence of saddle–node bifurcation in inflating and deflating processes, and the controllability of bifurcation by imposed voltage are verified. Zhu et al. [15] investigated the nonlinear oscillation of dielectric elastomer spherical balloon. The dielectric elastomer balloon resonates at multiple frequencies and there exist superharmonic, harmonic, and sub-harmonic responses. Furthermore, the jump phenomenon occurs at certain frequencies.

The early researches concentrate on the dynamical response of dielectric elastomer structures to harmonic or periodic excitation. The imposed excitations, however, are unavoidable perturbed by the fluctuation of electric voltage and mechanical pressure. The perturbation will severely deteriorate the performance of execution, such as the sensing and actuating precision for applications in

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sensors and actuators. Recently, the random responses of dielectric elastomer balloon to random voltage or random pressure have been investigated by Jin and Huang [16]. As the first work on the random aspect of dielectric elastomer structures, the constitutive relation is described by traditional neo-Hookean model and the analysis is executed in the vicinity of equilibrium position.

Although dielectric elastomer structures have many prominent advantages as listed above, many types of failure, such as electrical breakdown, electromechanical instability, loss of tension, and rupture by stretch may easily happen. Some works associated with various failure concentrate on the cases with quasi-static deformation [17,18]. In the aspect of dynamic stability, Chen et al. [19] investigated the dynamic electromechanical instability of dielectric elastomer balloon and gave the global critical condition. The random perturbation of electric voltage or mechanical pressure will induce dynamical response with larger amplitude, although with small probability. The response with large amplitude corresponds to larger strain and undoubtedly induces the failure happening more easily. This letter contributes to the ratio of expectation crossings of dielectric elastomer balloon subjected to random excitation, i.e., the probability of the stretch ratio exceeding a given threshold. It is appropriate to adopt the ratio of expectation crossing to evaluate system reliability.

The dielectric elastomer balloon keeps spherical symmetry and can be approximately simplified as a single-degree-of-freedom system. Denote the original radius and original thickness of the dielectric elastomer balloon in the initial state as R and K, respectively. In the operating state, a constant voltage Φ_0 is imposed on the two electrodes of the balloon, while a pressure difference between the internal and external surfaces $p = p_0(1+\xi)$ changes over time. p_0 is the mean value of the pressure difference, and $\xi(t)$ is random process described by Gaussian white noise with intensity $2D$. Thus, the instant radius r is time-varying and can be described by a random process. The initial state and operating state are shown in Fig. 1.

Suppose that the isothermal condition is kept under the dynamical process of the dielectric elastomer balloon. The thermodynamics of the dielectric elastomer balloon is characterized by the density of Helmholtz free energy, and the energy density is a sum of the elastic deformation energy and the dielectric energy. Herein, the Mooney–Rivlin model is adopted to describe the elastic deformation energy. The variation of the free energy equals the work done by the voltage, pressure, inertia force, and damping force. For simplicity, the viscous damping is adopted to describe the dissipative behavior. Then, the non-dimensional governing equation of the dielectric elastomer balloon is derived as [15,16]

$$\frac{\mathrm{d}^2\lambda}{\mathrm{d}t^2} + c\frac{\mathrm{d}\lambda}{\mathrm{d}t} + g(\lambda) = s_r\lambda^2\xi(t), \tag{1}$$

where $\lambda = r/R$ is the stretch ratio of the balloon, $t = \tilde{t}\sqrt{\mu/\rho}/R$ is the dimensionless time, c is the viscous damping coefficient, and

$$g(\lambda) = 2\lambda - 2\lambda^{-5} + 2\alpha(\lambda^3 - \lambda^{-3}) - s_r\lambda^2 - s_f\lambda^3, \quad s_r = \frac{p_0 R}{\mu K}, \quad s_f = \frac{2\varepsilon\Phi_0^2}{\mu K^2}, \tag{2}$$

in which, α is a material parameter, ρ is the density, μ is the shear modulus, ε is the permittivity of dielectric elastomer.

Integrating $g(\lambda)$ with respect to λ yields,

$$V(\lambda) = \lambda^2 + \lambda^{-4}/2 + \alpha\lambda^4/2 + \alpha\lambda^{-2} - s_r\lambda^3/3 - s_f\lambda^4/4, \tag{3}$$

which can be regarded as the system total potential. As the stretch ratio $\lambda = 0$ the total potential V is infinity and so there exists a singular point. The relation between the total potential and stretch ratio is shown in Fig. 2(a) for given s_r, s_f and several values of material parameter α. It can be seen that the total potential approaches a minimum V_m as $\lambda = \lambda_{\mathrm{eq}}$, and the curve is obviously asymmetric. For larger material parameter value only one potential well exists, while for smaller material parameter value the total potential will monotonically drop for large stretch ratio. The relation of total potential to stretch ratio is shown in Fig. 2(b) for $s_f = 0.2$, $\alpha = 0.2$ and several parameter values of s_r. The total potential possesses two stable wells as s_r in the range from 1.26 to 1.46. Out of this range, the total potential is mono-stable. This letter will focus on the mono-stable case while the bi-stable case will not be investigated.

The ratio of expectation crossings is the expectation of frequency of the stretch ratio crossing a given threshold. In order to derive the ratio of expectation crossings, the stationary joint probability density of stretch ratio and its ratio of change is first investigated. System in Eq. (1) is a strongly nonlinear stochastic dynamical system and it is hard to establish the exact solution of random response. In this letter, the stochastic averaging technique is adopted to derive the one-dimensional stochastic differential equation with respect to the system total energy, and then the stationary joint probabilistic density is given by solving the associated Fokker–Planck–Kolmogorov (FPK) equation.

Introduce the following transformation [20]

$$\mathrm{sgn}(\lambda - \lambda_{\mathrm{eq}})U(\lambda) = \sqrt{H}\cos\varphi, \quad \dot{\lambda} = -\sqrt{2H}\sin\varphi, \tag{4}$$

where the system total energy H is expressed as

$$H = \dot{\lambda}^2/2 + U(\lambda), \tag{5}$$

$$U(\lambda) = V(\lambda) - V_m, \tag{6}$$

and φ is the phase angle.

The stochastic differential equations for the system total energy H and the phase φ can be first derived. Then, with the application of the stochastic averaging technique, the system total energy $H(t)$ can be approximated by a one-dimensional diffusion equation and governed by the following Itô stochastic differential equation,

$$\mathrm{d}H = m(H)\mathrm{d}t + \sigma(H)\mathrm{d}B(t), \tag{7}$$

where $B(t)$ is a unit Wiener process, and the drift and diffusion coefficients are, respectively,

$$m = \frac{2}{T(H)}\int_{\lambda_1}^{\lambda_2}\left(-c\sqrt{2H-2U(\lambda)} + \frac{Ds_r^2\lambda^4}{\sqrt{2H-2U(\lambda)}}\right)\mathrm{d}\lambda, \quad \sigma^2 = \frac{2}{T(H)}\int_{\lambda_1}^{\lambda_2} 2Ds_r^2\lambda^4\sqrt{2H-2U(\lambda)}\mathrm{d}\lambda, \tag{8}$$

in which, λ_1 and λ_2 are the smallest and the largest roots of the equation $H - U(\lambda) = 0$, respectively, and the energy-dependent period is

$$T(H) = 2\int_{\lambda_1}^{\lambda_2}\frac{1}{\sqrt{2H-2U(\lambda)}}\mathrm{d}\lambda. \tag{9}$$

For the stationary state, the stationary probability density of system total energy $p(H)$ is governed by the following reduced FPK,

$$-\frac{\partial\left(m(H)p(H)\right)}{\partial H} + \frac{1}{2}\frac{\partial^2\left(\sigma^2(H)p(H)\right)}{\partial H^2} = 0. \tag{10}$$

The solution of Eq. (10) is

$$p(H) = \frac{C}{\sigma^2(H)}\exp\left(\int_0^H\frac{2m(u)}{\sigma^2(u)}\mathrm{d}u\right), \tag{11}$$

where C is a normalization constant. Then, the stationary joint probability density of the stretch ratio and its rate of change can be approximately calculated by

$$p(\lambda, \dot{\lambda}) = \left.\frac{p(h)}{T(h)}\right|_{h=\dot{\lambda}^2/2+U(\lambda)}. \tag{12}$$