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# An efficient and scalable approach for generating topologically optimized cellular structures for additive manufacturing

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ABSTRACT

This paper presents an end-to-end design process for compliance minimization-based topological optimization of cellular structures through to the realization of a final printed product. Homogenization is used to derive properties representative of these structures through direct numerical simulation of unit cell models. The resulting homogenized properties are then used assuming uniform distribution of the cellular structure to compute the macroscale structure. Results are presented that illustrate the fine-scale stresses developed in the macroscale optimized part as well as the effect that fine-scale structure has on the optimized topology. Finally, a new method is presented for generating an STL representation of the optimized part that is suitable for printing on typical industrial machines. Quite fine cellular structures are shown to be possible using this method.



## 1. Introduction

Interest in cellular materials continues to grow in lightweight applications as technologies to realize these materials become more reliable, repeatable and of lower cost. Besides the weight savings inherent in these materials due to their low density (relative to the solid base material), cellular structures can also exhibit good dynamic performance, defect tolerance, corrosion and thermal resistance and lower cost than traditional materials [1–3].

Following the taxonomy of Wadley [4] cellular structures fall into one of two broad categories; stochastic and periodic. Stochastic structures are those where the cellular structure is randomly distributed. While this cellular structure is among the easiest to manufacture using traditional techniques (e.g. foaming and sintering), its random layout and varying density make control of mechanical properties (and hence maximizing performance) difficult [4]. Better control of material properties are possible with periodic structures but their fabrication with traditional approaches is limited.

Periodic structures may also be amenable to numerical analysis at the scale of the part as homogenization techniques assume a regular or nearly regular structure at some scale [1]. Specifically, periodic cellular structures can be characterized by Representative Volume Elements (RVEs) that capture the repeated structure evident in the material. When the RVE is of a size much smaller than the part they compose (i.e., when "scale separation" exists), homogenization theory can be employed to provide representative material properties of the RVE that may be used to accurately describe the macroscale response of the structure.

Although conventional manufacturing approaches are available and dominate the creation of these cellular materials, their use becomes problematic for more complex cell geometries such as lattice structures [5]. These manufacturing limitations may be overcome by using Additive Manufacturing (AM) techniques. Unlike traditional subtractive processes, AM approaches "print" parts from, for example, powders or droplets that are fused together with thermal or other process. In this way, AM can build parts that are nearly impossible to realize with traditional subtractive processes due to (for example) geometric and material complexities accessible by these methods [6].

Additive manufacturing's promise of building novel parts is complemented by the Topology Optimization (TO) design paradigm. Topology optimization is a mathematical approach to

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optimize a part design given a set of constraints (e.g. loads, boundary conditions, mass and volume budget) [7,8]. Designs realized from TO are often non-intuitive with qualities that make them realizable only with additive manufacturing. The coupling of AM with TO has begun to realize significant attention [9] and examples of research in the area include [10,11,5,12].

Approaches to multi-scale topological optimization can be broadly classified as concurrent and non-concurrent. In the non-concurrent approach either the structural (macrostructure) or microstructure is optimized [5]. In the concurrent approach both macro- and microstructure are "designed" together to get a final part [13]. In this paper we consider the non-concurrent approach, selecting a (spatially) uniform microstructure and optimizing the macrostructure topology. Although the part design is based on the homogenized response, the method does permit examination of the total stresses (i.e. those that account for RVE stress raisers) in the final optimized geometries.

This paper begins with a discussion of homogenization theory and the application of homogenization to the linear elastostatic problem. This is followed by a description of the topological optimization algorithms used here. Note that a range of topological optimization algorithms are available for the compliance minimization problem including gradient-based methods such as Solid Isotropic Material with Penalization (SIMP), and level set techniques and evolutionary methods including the ESO and BESO approaches to name a few [7,8]. Here we choose the SIMP approach and describe some of the important aspects of our implementation. This discussion is followed by a description of our process for mapping the optimized topology to a faceted representation that incorporates the cellular structure explicitly (including a review of the current state of the art in this area). This leads directly to an STL file that may be exported to a 3D printer for production without further user intervention. Next, we present an example that illustrates the effect of fine-scale features on the macroscale design and total stress (i.e., micro- and macroscale). Finally, we present an example that demonstrates our process to design realizable cellular parts.

## 2. Problem formulation

In the linear elastostatic setting, the expression for static equilibrium (conservation of momentum) is given in terms of the displacement, $u_k(\mathbf{x})$, and elasticity tensor, $E_{ijkl}$, as:

$$\frac{\partial}{\partial x_j}\left(E_{ijkl}\left(\frac{\partial u_k}{\partial x_l}+\frac{\partial u_l}{\partial x_k}\right)\right)=0 \quad \text{in} \quad \Omega \tag{1}$$

$$E_{ijkl}\left(\frac{\partial u_k}{\partial x_l}+\frac{\partial u_l}{\partial x_k}\right)\hat{n}_j=t_i^o \quad \text{on} \quad \partial\Omega^t \tag{2}$$

$$u_i=u_i^o \quad \text{on} \quad \partial\Omega^u \tag{3}$$

where $t_i^o$ is the applied traction, $u_i^o$ is the displacement constraint, $\hat{n}_j$ is the boundary surface normal, $\partial\Omega^u \cup \partial\Omega^t = \partial\Omega$, and $\partial\Omega$ is the boundary of the domain, $\Omega$.

In Sections 2.1 and 2.2 below, a brief review of homogenization theory and the numerical solution of the resulting equations is provided. For a more thorough treatment of homogenization theory in the context of topology optimization, see Hassani and Hinton [1].

### 2.1. Homogenization

In periodic or nearly periodic heterogeneous materials the elasticity tensor is assumed to vary with the period of the structure, $\epsilon$. The goal of homogenization is to (a) determine *effective* or *homogenized* material constants that account for this microscale variation, and (b) provide a means for examining the local or microscopic fields in component scale analysis [14,1]. The approach begins with an asymptotic expansion of the dependent variable in the period of the structure, $\epsilon$,

$$u_k(\mathbf{x}) \approx u_k^0\left(\mathbf{x},\frac{\mathbf{x}}{\epsilon}\right)+\epsilon u_k^1\left(\mathbf{x},\frac{\mathbf{x}}{\epsilon}\right)+\epsilon^2 u_k^2\left(\mathbf{x},\frac{\mathbf{x}}{\epsilon}\right)+\cdots \tag{4}$$

Superscripts on the displacements, $u_k$, are indices, not exponents. The material response is assumed to be $Y$-periodic, where $Y$ is the domain of the periodic cell. Defining the local variable, $\mathbf{y}=\mathbf{x}/\epsilon$, and recasting Eq. (1) in terms of the expansion yields

$$\mathcal{A}_{ik}^{\epsilon}u_k=0 \tag{5}$$

where $\mathcal{A}_{ik}^{\epsilon}$ is the differential operator:

$$\mathcal{A}_{ik}^{\epsilon}=\frac{\partial}{\partial x_j}\left(E_{ijkl}^{\epsilon}(\mathbf{y})\frac{\partial}{\partial_s x_l}\right), \tag{6}$$

$E_{ijkl}^{\epsilon}(\mathbf{y})$ is the periodic elasticity tensor defined on the periodic domain, and the subscript 's' indicates a symmetric gradient, i.e.,

$$\frac{\partial u_k}{\partial_s x_l}=\frac{1}{2}\left(\frac{\partial u_k}{\partial x_l}+\frac{\partial u_l}{\partial x_k}\right). \tag{7}$$

Expanding Eq. (5) yields the more convenient form,

$$\mathcal{A}_{ik}^{\epsilon}=\frac{1}{\epsilon^2}\mathcal{A}_{ik}^1+\frac{1}{\epsilon}\mathcal{A}_{ik}^2+\mathcal{A}_{ik}^3, \tag{8}$$

in terms of the differential operators

$$\mathcal{A}_{ik}^1=\frac{\partial}{\partial y_j}\left(E_{ijkl}^{\epsilon}(\mathbf{y})\frac{\partial}{\partial_s y_l}\right) \tag{9}$$

$$\mathcal{A}_{ik}^2=\frac{\partial}{\partial y_j}\left(E_{ijkl}^{\epsilon}(\mathbf{y})\frac{\partial}{\partial_s x_l}\right)+\frac{\partial}{\partial x_j}\left(E_{ijkl}^{\epsilon}(\mathbf{y})\frac{\partial}{\partial_s y_l}\right) \tag{10}$$

$$\mathcal{A}_{ik}^3=\frac{\partial}{\partial x_i}\left(E_{ijkl}^{\epsilon}(\mathbf{y})\frac{\partial}{\partial_s x_l}\right). \tag{11}$$

Eq. (5) is satisfied if terms on powers of $\epsilon$ are equal to zero, i.e.,

$$\mathcal{A}_{ik}^1u_k^0=0 \tag{12}$$

$$\mathcal{A}_{ik}^1u_k^1+\mathcal{A}_{ik}^2u_k^0=0 \tag{13}$$

$$\mathcal{A}_{ik}^1u_k^2+\mathcal{A}_{ik}^2u_k^1+\mathcal{A}_{ik}^3u_k^0=0. \tag{14}$$

Eqs. (12)–(14) correspond to the lowest order terms, $\epsilon^{-2}$, $\epsilon^{-1}$, and $\epsilon^0$, respectively. Since we are concerned with the limit as $\epsilon\to 0$, any higher order terms are neglected.

Eq. (12) requires $u_k^0$ be constant in $\mathbf{y}$, i.e., $u_k^0=u_k^0(\mathbf{x})$, so Eq. (13) can be reduced to

$$\mathcal{A}_{ik}^1u_k^1=-\frac{\partial E_{ijkl}^{\epsilon}}{\partial y_j}\frac{\partial u_k^0}{\partial_s x_l}. \tag{15}$$

At this point we wish to find a solution for the microscale displacement, $u_k^1$, to Eq. (15) that can then be used to reduce Eq. (14) to include only macroscale terms. The reduced form will define the homogenized problem that implicitly accounts for features at the microscale. To that end, we define the "cell problem" in the domain, $Y$, of the periodic cell to be

$$\mathcal{A}_{ij}^1\chi_j^{kl}=\frac{\partial E_{ijkl}^{\epsilon}}{\partial y_j} \tag{16}$$

where the characteristic displacements, $\chi_j^{kl}(\mathbf{y})$, are $Y$-periodic. Combining the cell problem with Eq. (15) yields a solution for the fine scale:

$$u_i^1(\mathbf{x},\mathbf{y})=-\chi_i^{kl}(\mathbf{y})\frac{\partial u_k^0}{\partial_s x_l}+\tilde{u}_i(\mathbf{x}) \tag{17}$$

where $\tilde{u}_i(\mathbf{x})$ is an arbitrary additive constant.

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