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Evaluation of flexural failure of sill mats using classical beam theory and numerical models

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A R T I C L E I N F O

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1. Introduction

Sill mats are structural elements that provide support in underground mining (Fig. 1). In vertical or sub-vertical orebodies, an ore sill pillar is replaced by a sill mat counterpart when the economic benefit is positive, such as when the ore grade is enough to make positive earnings. A sill mat is constructed using multiple batches of cemented rock fill or pastefill (cemented tailings) placed into the previously mined ore sill pillar to a predefined height derived from its design. Sill mats may or not be anchored to the hanging and foot walls, and usually are sufficiently long in the longitudinal direction making a two-dimensional analysis appropriate to be performed. When the underlying stope is removed, the load of the unconsolidated backfill that rests on top of the sill mat and its self-weight must be supported by this structural element. Therefore, a correctly designed sill mat should be stable under these loads, allowing the safe operation of mining personnel and equipment underneath it. The proper design of these mining structures requires the correct determination of the vertical stresses to be supported by the sill mat and the structural capacity required to withstand these loads.

In practice, sill mats are constructed based on two main approaches; experience, and engineering principles. From an engineering point of view, it has been considered beam theory or solid mechanics theory, numerical modelling, physical modelling (centrifuge modelling in some cases).[1–6] Analytical theory consider sill mats as structural elements that may fail under sliding, flexural, rotation, or caving modes of failure.[1] One of the main types of failure is the flexural failure, and the deduction of its analytical equation considers classical beam theory applied to this mining problem.[2] Further in this work is examined the applicability of this theory to this mining design.

Data of actual geometry and material properties of sill mats used in mining can be found in 3,4,6,7 and 8 and are shown in Table 1.

2. Analytical interpretation of flexure failure based on classical beam design

Vertical load (σ_v) causes the sill mat to bend, meaning that the upper surface must be shorter than the lower surface or vice versa depending of the applied load. Therefore, the strain are different along the sill mat, and because the stress is directly proportional to strain, it follows that the stress will vary through the depth of the mat. This bending creates a compressive stress in the upper portion and a traction in the lower portion of the sill mat (or vice versa depending on location). In between, there is a plane where its elements are neither stretched nor compressed, called the neutral plane. Since the constructing materials used to fabricate the sill mats have larger compressive strength than tensile strength, failure starts when the tensile stresses reach the ultimate tensile strength of the material (σ_t).

As mentioned previously, flexural failure considerations using beam theory provides the designer with a simple tool to analyze sill mat stability. In 1 is considered that "a wide sill mat would, quite obviously, be susceptible to flexural failure due to the relatively low tensile strength of cemented tailings". He proposed the standard formulae for a fixed end uniformly loaded beam (as shown in Fig. 2).

Considering the maximum moment (M_{max}) generated at both ends (Fig. 2a), Mitchell proposed that failure occurs when the maximum traction on the sill mat (σ_{max}) reaches the maximum tensile strength (σ_t), $\sigma_t = \sigma_{max}$.

When solving the value of σ_{max}, generated by the vertical uniform

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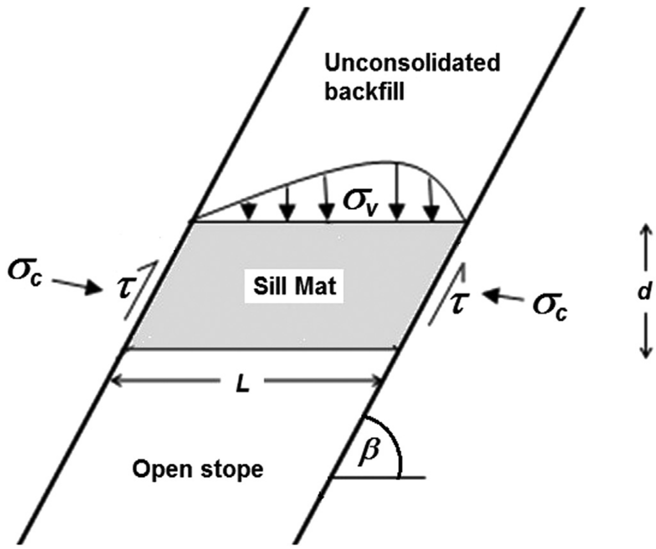

Fig. 1. Schematic representation of sill mats.

load σ_v (Figs. 1 and 2) under the adequate boundary conditions, it will be established that the geometrical condition for flexure failure will initiate at both ends of the beam, as it is shown in Fig. 2b, is given by:

$$\left(\frac{L}{d}\right)^2 > 2(\sigma_t + \sigma_c)/\sigma_v \tag{1}$$

where σ_t is the tensile strength of the sill mat and σ_v is the uniform loading which should include its self-weight. σ_c is the compressive stress applied to both ends of the beam. Thereafter, failure by traction is observed at the center of the beam as it is shown in Fig. 2c.

Eq. (1) is deduced using the Euler-Bernoulli bending theory or simple beam theory. The use of this theory involves the following hypotheses. (a) The load acting is normal to its surface. (b) Deflections are small in comparison with the thickness of the beam/plate (or Sill Mat). (c) The cross section is assumed to remain perpendicular to the axis direction. (d) The weight of the beam/plate can be included in the load σ_v. (e) Also, the problem is considered as a state of plane stress, in which the normal stress σ_z, and the shear stresses σ_{xz} and σ_{yz} are also assumed to be zero. According to the hypothesis mentioned before, the general beam equation will be deduced in the following paragraphs.

Fig. 2 shows a cross section of the beam where there is a zone in traction and the opposite side in compression. In between, there is a neutral axis, where there is no axial deformation. Considering "u_z" $= w$ as the vertical deflection of the neutral axis, the deformation in the cross section of the beam in the x direction is given by:

$$u_x = -z\psi(x) \tag{2}$$

As the plane AB remains perpendicular to CD (Euler-Bernoulli assumption) then:

$$\psi = dw/dx \tag{3}$$

Then

$$u_x = -z\frac{dw}{dx} \tag{4}$$

and

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x} = -z\frac{d^2w}{dx^2} \tag{5}$$

Then assuming $\sigma_{yy} = \sigma_{zz} = \sigma_{xy} = \sigma_{yz} = 0$ the stress-strain relationships give

$$\varepsilon_{xx} = \frac{1}{E}[\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz})] = \frac{1}{E}\sigma_{xx} \tag{6}$$

$$\sigma_{xx} = -Ez\frac{d^2w}{dx^2} \tag{7}$$

Then for equilibrium of momentum:

$$M = -\int_A \sigma_{xx} z dA = E\frac{d^2w}{dx^2}\int_A z^2 dA = EI\frac{d^2w}{dx^2} \tag{8}$$

$$\Rightarrow \sigma_{xx} = -Mz/I \tag{9}$$

where I is the second moment of inertia, and z is the distance from the neutral axis of the beam.

Additionally,

$$\frac{dM}{dx} = V(x),\ \frac{dV}{dx} = \sigma_z(x),\ EI\frac{d^4w}{dx^4} = \sigma_z(x) \tag{10}$$

where σ_z is the applied force/unit length on the beam in the z direction (or σ_v for this example).

Solving the above equations for the corresponding boundary conditions, the deflection w, the shear stress V, and the moment M on a beam with both ends fixed can be obtained. For this analysis the sill mat is considered with both ends fixed (as shown in Fig. 2), then it becomes a statically indeterminate beam problem. Therefore to obtain the solution for this beam the static equilibrium is used and considering known values of slope and deflection at particular beam sections, the solution for maximum moment located at both ends is given by[9]:

$$\mathrm{M}_{\max} = \sigma_v L^2/12 \tag{11}$$

and the moment at the center of the beam is:

$$\mathrm{M}_{\max} = \sigma_v L^2/24 \tag{12}$$

Consequently the maximum traction will be located at both ends and has a value derived from Eq. (1):

$$\sigma_{max} = \frac{M_{max} * \frac{d}{2}}{\frac{1}{12}d^3} = \frac{\frac{\sigma_v L^2}{12}\cdot\frac{d}{2}}{\frac{1}{12}d^3} = \frac{\sigma_v L^2}{2d^2} \tag{13}$$

Failure will initiate on both ends of the beam (or sill mat). Once these two points show plastic deformation, the center of the beam will start failing as shown in Fig. 2.

Considering that an additional compressive force is applied to the beam (σ_c) the maximum applied traction is: (σ_{max} - σ_c) and in order to be stable, the tensile strength σ_t has to be greater than σ_{max} - σ_c, then Eq. (1) can be deduced directly in 1.

In many simulations, the evaluation of flexure failure has been performed using this simple equation. However, in sill mat design the problem is closer to a plane strain than a plane stress approximation. This is due the fact that the longest dimension of the sill mat is much larger than the span. Additionally, the other hypothesis used to derive this equation will be discussed further after numerical model results are shown. More details about Euler-Bernoulli beam theory can be found in 9–11.

3. Numerical modelling approach for sill mats

FLAC 2D version 7 was the numerical modelling code employed for modelling and estimating the factor of safety of different sill mat geometries under varying strength properties. FLAC uses a finite difference approach that allows the use of different constitutive models to represent rock mass or any other material as a continuum in order to determine its behavior under the loads being applied. Fig. 3 shows the mesh used in the numerical model. The dimensions of the elements of the mesh is maintained constant independently of the size of the modelled sill mat. The unit weight of the backfill material was imposed at very low value since the vertical load σ_v takes into consideration the self-weight of the sill mat.

Only flexure resistance of the sill mat (tensile failure) will be considered in this analysis. Therefore, shear strength will be imposed at