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Extracting shear and normal compliances of crack-like defects from pressure dependences of elastic-wave velocities

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ABSTRACT

We present a differential formulation of effective-medium model in which the normal and shear compliances of the high-compliance porosity are explicitly decoupled. This feature of the decoupled-compliance model (DC model) is in contrast to conventional models in which such defect's properties are implicitly assumed and are subject to strong limitations defined by the used particular crack model. Comparison with the DC model makes it possible to reveal such implicit assumptions in the conventional models. Furthermore, for the conventional cracks, our approach gives the same results as the conventional models. The ability of the DC model to incorporate arbitrary defect properties in terms of their normal-to-shear compliance ratio (*q*-ratio) is used to formulate an analogue of Hashin-Shtrikman constraints on the range of feasible crack-induced variations in the moduli. Comparison of the DC model with experimental pressure dependences of elastic-wave velocities in rocks makes it possible to extract the *q*-ratio for real crack-like defects. These results demonstrate that properties of real cracks usually noticeably differ from those of popular crack models such as cracks with free faces (e.g., penny-shape) or pure shear cracks. We discuss an example of sandstone with pronouncedly negative Poisson's ratio that is due to the fact that the ratio of normal-to-shear compliances of voids in this rock (q~7–8) is significantly higher than for the conventional cracks (q~2). Ability of the DC model to accurately extrapolate pressure dependences of the moduli from relatively low pressures to several times greater is demonstrated, including the cases, for which the conventional models give huge errors. The introduced parameter q – the ratio of normal-to-shear compliances of voids – provides additional insight into properties of real crack-like defect in rocks.

1. Introduction

Investigation of wave propagation in rocks is an important tool in analyzing rock microstructure and its evolution under loading. A key element in it is the possibility of interpreting the measured wave velocities in terms of concentration of internal voids (cracks, pores) and their distributions. In many cases variations of total rock porosity are fairly small and wave velocities are mostly determined by variations in the effective elastic moduli. This makes the use of variations in effective elastic moduli with the load a valuable source of information.

It is generally accepted that in terms of sensitivity to applied pressure, it is reasonable to divide the total porosity into "stiff" pores and "high-compliance" ones.[1] The stiff porosity comprises spheroidal and cylinder-like pores (often called "equant" porosity). The volume fraction of such pores only weakly changes under pressures up to the strength limit of the material. Thus the realistic pressures below the damage limit insignificantly affect the stiff-porosity contribution to variations in the elastic moduli. In contrast, the content of compliant crack-like pores may already be very strongly changed by much lower and attainable pressures causing rather significant variations in the elastic moduli. The well known "rule of thumb"[2] states that thin crack-like pores with small aspect ratio (i.e., the ratio between the crack opening and its characteristic diameter) can be completely closed by creating in the material the average strain approximately equal to the crack's aspect ratio. For example, in sandstones pressures ~10^2 MPa can close all cracks with aspect ratios below 10^{-2}–10^{-3}, while equant pores with significantly larger aspect ratios can remain open (see e.g., examples discussed in 3).

Besides the difference in the stress-sensitivity, in the context of variations in different elastic moduli induced by anisotropic-in-shape high-compliance crack-like pores, the key feature of the latter is the value of the ratio between the crack compliance with respect to applied normal and tangential (shear) stresses. In what follows we call this quantity "*q*-ratio". This ratio is the main parameter controlling the

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proportion between crack-induced variations in different elastic moduli. Some limiting cases of this effect were understood fairly long ago. For example, it was clearly understood (see e.g. 4) that the Poisson's ratio *increases* due to the presence of liquid-saturated (wet) cracks, for which the compressibility with respect to the normal applied stress is strongly reduced, whereas their shear (lateral) compliance remain weakly affected by the liquid. In contrast, for the same rock in dry state, the presence of dry cracks with much greater normal compliance that in the wet state leads to *decrease* in the Poisson's ratio. Certainly, cracks-like defects with intermediate properties may produce intermediate effects.[4]

Qualitatively the decrease in the Poisson's ratio due to dry cracks agrees with some widely used effective-medium models[4]; however, even a quick glimpse at available in literature data on pressure-dependences of wave velocities in dry crack-containing rocks (see, e.g., examples in[3]) indicates that complementary variations in P- and S-wave velocities can differ rather significantly. This observation clearly indicates that even for dry cracks, it is not *a priori* given that the ratio of normal-to shear compliance always is the same, although in the conventionally used models this fact is often not explicitly mentioned (see, e.g., the well-known model[4] based on penny-shape cracks and widely used reformulations of this model into differential form discussed in books of Zimmerman,[5] Jaeger, Cook and Zimmerman,[6] and Mukerji, Mavko and Dvorkin[7]). Although contributions of cracks to normal and shear compliances were discussed quite long ago, e.g., by Kachanov,[8,9] these widely accepted models intrinsically are not adapted for inferring elastic properties of real high-compliant crack-like porosity from experimental data.

In this paper, we consider how elastic properties of real high-compliance voids can be extracted from experimental data, including the cases of rather significant variations in the elastic moduli that are beyond the applicability limits of simple small-perturbation approximation. Our starting point is the effective-medium model[10] with explicitly decoupled compliances (DC model) of compliant crack-like pores with respect to normal and shear loading. Unlike popular models such as[4] or its modifications[5,6] based on rather specific assumptions about crack-like defects like penny-shape ones, the main feature of approach[8] was the absence of any *a priori* imposed limitation on the ratio between the normal and shear compliance of soft pores. In a broad sense a similar approach was used in some models of elasticity of granular materials formulated in terms of normal and tangential stiffness of intergrain contacts,[11] which allowed for estimating the ratio between the contact-stiffness parameters from comparison with experimental data for granular materials.[12]

The initial form of DC model[8] was formulated in the no-interaction approximation (asymptotics of low void concentrations); later the same approximation was used in 13 to obtain exactly the same (with some difference in the notations) equations for reduction of the bulk and shear moduli expressed via explicitly decoupled total normal- and shear compliances imparted to the rock mass. However, in 11 the discussion of various experimental data on pressure dependences of elastic-wave velocities was focused on the differences in pressure sensitivity of various rocks rather than on the differences of elastic properties of crack-like pores in terms of their normal and shear compliances (i.e., their q-ratio).

A discussion on the differences of elastic properties of crack-like voids was presented in paper 14 where DC model[8] with explicitly decoupled compliances of cracks was applied to some literature data on experimentally measured pressure-dependences of P- and S-wave velocities in rocks. This examination revealed examples of rather unexpected properties of dry crack-like porosity with anomalously high ratios between normal and shear compliances, which very significantly differ from properties of cracks used in majority of conventional models.[5,6] In particular, the fact of the presence of negative values of the Poisson's ratio for sufficiently high concentration of cracks with predominantly normal compliance was revealed in 12 This finding agrees with conclusions about the possibility of negative Poisson's ratio in rocks with randomly oriented cracks exhibiting predominantly normal compliance,[8] including the ultimate case of purely normally compliant cracks also discussed in 15,16. In the latter case, the minimal attainable Poisson's ratio is $-1/3$.[8,13,14]

However, conclusions[12] (including unusually high ratios of normal-to-shear compliance and especially the revealed fact of negative Poisson's ratio) were based on the application of DC model[8] derived in the no-interaction approximation, whereas the reduction in the moduli for the examined rocks was quite significant (tens per cent) and evidently corresponded to rather high crack concentrations, for which applicability of the no-interaction approximation becomes questionable. This observation indicates the necessity of more rigorous reformulation of the decoupled-compliances approach to account for finite concentrations of cracks.

Presently, there is widely accepted understanding (see, e.g.,[5,6]) that among various analytical approaches used for accounting for finite concentrations of porosity, the so-called differential scheme (implying portion-by-portion addition of the voids for calculation of moduli reduction) is more adequate in comparison with other ones (such as algebraic “self-consistent” approach used in 4).

Therefore, in what follows we reformulate the initial no-interaction version of DC model[8] in the differential form and obtain its analytical solutions. Then using the rigorous decoupled-compliance differential model (DCD model) we demonstrate that despite the fact that the effect of cracks on the moduli reduction may be quantitatively underestimated in the no-interaction approximation, the latter can still predict well the proportion of the complementary variations in the P- and S-wave velocities (or other independent elastic moduli). Furthermore, the reformulation of the DC model[8] in the differential form has confirmed the validity of the earlier obtained conclusions[12] about unusual elastic properties of cracks and provided a new criterion of their verification. The performed re-examination and interpretation of other experimental data from different sources consistently demonstrate that elastic properties of cracks in many cases are significantly different from those implied in widely used conventional models and thus may strongly affect interpretation of seismic data in such cases.

2. Elastic properties of cracks implicitly assumed in conventional models

To better illustrate the essence of the implicit, but conventionally used assumptions about the ratio between the normal and shear compliance of crack-like voids one may take the expressions for the effective bulk and shear elastic moduli K_{eff} and G_{eff} in the small-perturbation limit retaining only linear terms with respect to the crack concentration. In this lowest-order approximation, the no-interaction, differential or self-consistent forms of the effective-medium model reduce to the same form, so that the choice of a particular model is not critical for understanding the assumed crack properties. It is convenient to choose the expressions for effective moduli reduction used by Salganik[17] and Budiansky and O'Connell[4] for a solid with isotropically oriented dry penny-shape cracks in the asymptotics of small concentration (only terms linear with respect to concentration are retained). Denoting the corresponding effective bulk and shear moduli affected by penny-shape cracks as K_{PSC} and G_{PSC} we obtain:

$$\frac{K_{PSC}}{K_0} = \frac{1}{1 + \frac{16}{9}\frac{1-\nu_0^2}{1-2\nu_0}\Gamma} \approx 1 - \frac{16}{9}\frac{1-\nu_0^2}{1-2\nu_0}\Gamma + O(\Gamma^2), \tag{1}$$

$$\frac{G_{PSC}}{G_0} = \frac{1}{1 + \frac{32}{45}\frac{(1-\nu_0)(5-\nu_0)}{2-\nu_0}\Gamma} \approx 1 - \frac{32}{45}\frac{(1-\nu_0)(5-\nu_0)}{2-\nu_0}\Gamma + O(\Gamma^2), \tag{2}$$

where ν is the Poisson's ratio and the subscript “0” relates to the unperturbed parameters of the solid matrix. The effective concentration of cracks Γ according to [4] has the following meaning: