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Uniqueness of displacement back analysis of a deep tunnel with arbitrary cross section in transversely isotropic rock

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1. Introduction

The first paper about displacement back analysis written by Kavanagh and Clough was published on International Journal of Solids and Structures.[1] After that, the research on displacement back analysis began to develop quickly. After more than 40 years of development, a lot of corresponding results were obtained.[2–12]

The researches on displacement back analysis were extended from the practical to the theoretical and they have been widely applied to identify the ground stress and parameters of the rock. The rock parameters of underground caverns and tunnels were estimated based on the back analysis results,[13–19] and some researches are focus on the soft and weak rock.[14,20] The back analysis method was also used to the analyze the stability of slope engineering.[21,22] The intelligent back analysis methods was developed and used in many practical engineering.[23–26] Back analysis method was also combined with other techniques such as AE.[27]

The back analysis method has been widely developed and used, however, there were issues of uniqueness of the inversed parameters in the studies of displacement back analysis form the outset. The research on uniqueness of displacement back analysis was one of the most important but least well-studied issues, which is how to select the appropriate unknown parameters to avoid the multiple solutions of the inversion results. Using identifiability condition, Lu explored the identifiability of surrounding rock parameters and initial stress, and got the result that dispalcement back analysis can not identify all the parameters of surrounding rock and initial stress.[28] The uniqueness of dispalcement back analysis of plane linear elastic problem in a tunnel with arbitrary cross section was discussed by Zhang 29. The results showed that inversion parameters were more restrictive in a tunnel with simple shapes. Simultaneously, the possibility of uniqueness was small when a plurality of parameters were inversion. Yang discussed the uniqueness of the solution determined by the TBA method.[30] According to the ANFIS method, Ding obtained the uniqueness of inversion results.[31]

In all the above literatures, the rock was assumed as isotropic material. However, because of the anisotropy, the rock will exhibit different deformation characteristics, different elastic modulus and Poisson's ratio as loaded in different directions. While the researches on uniqueness of displacement back analysis in anisotropic rock were rare. Based on the above researches, this paper focuses on the uniqueness of displacement back analysis in transversely isotropic rock. Considering the no uniqueness shape of underground tunnels, the uniqueness of displacement back analysis of a deep tunnel with arbitrary cross section in transversely isotropic rock is studied firstly, then a circular tunnel example was presented to demonstrate the procedure and method shown in this paper.

2. Parameter identifiability condition

According to the necessary and sufficient conditions which make the objective function minimum, Lu derived the parameter identifiability condition.[28] In which η_i was assumed as the i-th displacement output, and β_i was assumed as the j-th inversion parameter. Then $\partial\eta_i/\partial\beta_i$ was defined as the sensitive coefficient. Within the measurement limitations, if the sensitive coefficients of all the unknown parameters are linear independence, then all these parameters can be uniquely identified by the measured displacement values, namely the back analysis is unique. Conversely, if the sensitive coefficients are linear correlation, then all these parameters cannot be uniquely identified by the measured displacement values, namely the back analysis is not unique.

Whether the sensitive coefficients are linear correlation or linear independence can be judged according to the following:

$$C_1\frac{\partial\eta_i}{\partial\beta_1}+C_2\frac{\partial\eta_i}{\partial\beta_2}+\cdots+C_m\frac{\partial\eta_i}{\partial\beta_m}=0,\ i=1,2,\ldots,l \tag{1}$$

Where $\beta_1, \beta_2, \ldots, \beta_m$ are inversion parameters, $i = 1, 2, \cdots l$ is the number of the measuring points.

Linear correlation of the sensitive coefficients means that there is at least one parameter C_i is not equal to zero to hold the Eq. (1) when the position output $i = 1, 2, \ldots, l$. Linear independence of the sensitive coefficients means that only if all the parameters C_i are equal to zero when the position output $i = 1, 2, \ldots, l$, then the Eq. (1) can be set up.

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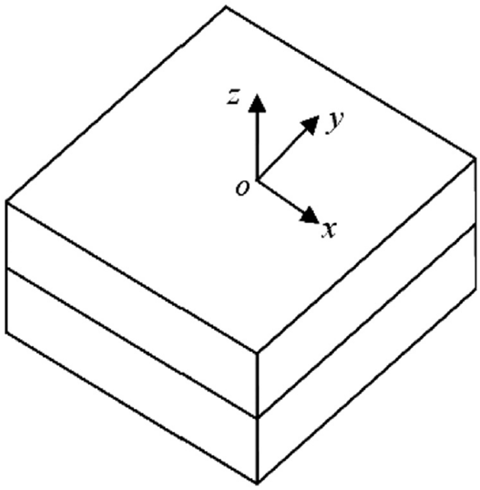

Fig. 1. Model of a transversely isotropic solid.

The uniqueness of displacement back analysis can be obtained after the linear correlation or linear independence of the sensitive coefficients are confirmed.

3. Displacement analytical solutions for a deep tunnel with arbitrary cross section in transversely isotropic rock

A model of transversely isotropic solid is shown in Fig. 1, in which the plane xoy is regarded as the plane of isotropy and oz is the axis of material symmetry.

Ideal model of a deep tunnel with arbitrary cross section in transversely isotropic rock is shown in Fig. 2. In which The tunnel is straight and its axis coincides with the axis of the xoy plane.

Displacement analytical solution for a deep tunnel with arbitrary cross section in a transversely isotropic rock is.[32]

$$\begin{aligned} u_r = {} & (\frac{3-\mu}{E} - \frac{4\mu'^2}{E'})\{\frac{p+q}{2}R_e[F_1(z)] + \frac{p-q}{2}R_e[F_2(z)]\}\cos\theta \\ & + \frac{1+\mu}{E}\{\frac{p+q}{2}R_e[F_3(z)] + \frac{p-q}{2}R_e[F_4(z)]\}\cos\theta \\ & + (\frac{3-\mu}{E} - \frac{4\mu'^2}{E'})\{\frac{p+q}{2}I_m[F_1(z)] + \frac{p-q}{2}I_m[F_2(z)]\}\sin\theta \\ & + \frac{1+\mu}{E}\{\frac{p+q}{2}I_m[F_3(z)] + \frac{p-q}{2}I_m[F_4(z)]\}\sin\theta \end{aligned} \tag{2}$$

where μ_r, p and q are respectively radial displacement, initial ground

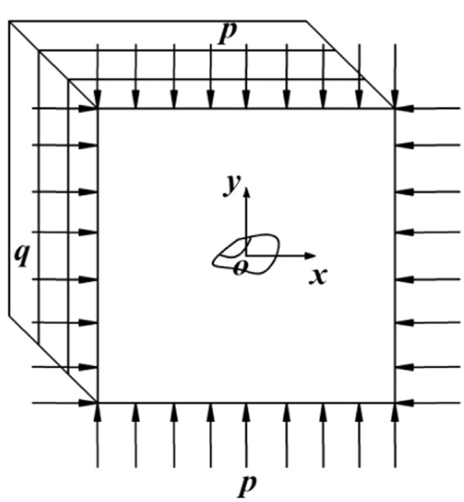

Fig. 2. Ideal model of a deep tunnel with arbitrary cross section in transversely isotropic rock.

stress in the vertical direction and horizontal direction, E and μ are respectively the elastic modulus and Poisson ratio in the isotropic plane, E' and μ' are respectively the elastic modulus and Poisson ratio in the direction normal to the isotropic plane, θ is the inclination in the polar coordinates, $F_1(z)$~$F_4(z)$ are plural and they are functions of coordinates, which are related to the shape of the tunnel but not related to the physical parameters, The functions $F_1(z)$~$F_4(z)$ have definite forms for each established tunnel with arbitrary cross section.

4. Uniqueness of displacement back analysis of a deep tunnel with arbitrary cross section in transversely isotropic rock

4.1. Discussion of uniqueness of displacement back analysis

There are six physical parameters in the Eq. (2), which are p,q,E,μ,E' and μ'. Sensitive coefficient for each parameter can be obtained as the following:

$$\begin{aligned} \frac{\partial u_r}{\partial p} = {} & (\frac{3-\mu}{E} - \frac{4\mu'^2}{E'})\{\frac{1}{2}R_e[F_1(z)] + \frac{1}{2}R_e[F_2(z)]\}\cos\theta \\ & + \frac{1+\mu}{E}\{\frac{1}{2}R_e[F_3(z)] + \frac{1}{2}R_e[F_4(z)]\}\cos\theta \\ & + (\frac{3-\mu}{E} - \frac{4\mu'^2}{E'})\{\frac{1}{2}I_m[F_1(z)] + \frac{1}{2}I_m[F_2(z)]\}\sin\theta \\ & + \frac{1+\mu}{E}\{\frac{1}{2}I_m[F_3(z)] + \frac{1}{2}I_m[F_4(z)]\}\sin\theta \end{aligned} \tag{3}$$

$$\begin{aligned} \frac{\partial u_r}{\partial q} = {} & (\frac{3-\mu}{E} - \frac{4\mu'^2}{E'})\{\frac{1}{2}R_e[F_1(z)] - \frac{1}{2}R_e[F_2(z)]\}\cos\theta \\ & + \frac{1+\mu}{E}\{\frac{1}{2}R_e[F_3(z)] - \frac{1}{2}R_e[F_4(z)]\}\cos\theta \\ & + (\frac{3-\mu}{E} - \frac{4\mu'^2}{E'})\{\frac{1}{2}I_m[F_1(z)] - \frac{1}{2}I_m[F_2(z)]\}\sin\theta \\ & + \frac{1+\mu}{E}\{\frac{1}{2}I_m[F_3(z)] - \frac{1}{2}I_m[F_4(z)]\}\sin\theta \end{aligned} \tag{4}$$

$$\begin{aligned} \frac{\partial u_r}{\partial E} = {} & -\frac{3-\mu}{E^2}\{\frac{p+q}{2}R_e[F_1(z)] + \frac{p-q}{2}R_e[F_2(z)]\}\cos\theta \\ & - \frac{1+\mu}{E^2}\{\frac{p+q}{2}R_e[F_3(z)] + \frac{p-q}{2}R_e[F_4(z)]\}\cos\theta \\ & - \frac{3-\mu}{E^2}\{\frac{p+q}{2}I_m[F_1(z)] + \frac{p-q}{2}I_m[F_2(z)]\}\sin\theta \\ & - \frac{1+\mu}{E^2}\{\frac{p+q}{2}I_m[F_3(z)] + \frac{p-q}{2}I_m[F_4(z)]\}\sin\theta \end{aligned} \tag{5}$$

$$\begin{aligned} \frac{\partial u_r}{\partial \mu} = {} & -\frac{1}{E}\{\frac{p+q}{2}R_e[F_1(z)] + \frac{p-q}{2}R_e[F_2(z)]\}\cos\theta \\ & + \frac{1}{E}\{\frac{p+q}{2}R_e[F_3(z)] + \frac{p-q}{2}R_e[F_4(z)]\}\cos\theta \\ & - \frac{1}{E}\{\frac{p+q}{2}I_m[F_1(z)] + \frac{p-q}{2}I_m[F_2(z)]\}\sin\theta \\ & + \frac{1}{E}\{\frac{p+q}{2}I_m[F_3(z)] + \frac{p-q}{2}I_m[F_4(z)]\}\sin\theta \end{aligned} \tag{6}$$

$$\begin{aligned} \frac{\partial u_r}{\partial E'} = {} & \frac{4\mu'^2}{E'^2}\{\frac{p+q}{2}R_e[F_1(z)] + \frac{p-q}{2}R_e[F_2(z)]\}\cos\theta \\ & + \frac{4\mu'^2}{E'^2}\{\frac{p+q}{2}I_m[F_1(z)] + \frac{p-q}{2}I_m[F_2(z)]\}\sin\theta \end{aligned} \tag{7}$$

$$\begin{aligned} \frac{\partial u_r}{\partial \mu'} = {} & -\frac{8\mu'}{E'}\{\frac{p+q}{2}R_e[F_1(z)] + \frac{p-q}{2}R_e[F_2(z)]\}\cos\theta \\ & - \frac{8\mu'}{E'}\{\frac{p+q}{2}I_m[F_1(z)] + \frac{p-q}{2}I_m[F_2(z)]\}\sin\theta \end{aligned} \tag{8}$$

Taking the sensitive coefficients of the individual parameters into the following formula: