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A constitutive model based on the modified generalized three-dimensional Hoek–Brown strength criterion

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ABSTRACT

A new constitutive model is proposed based on the generalized three-dimensional (3D) Hoek-Brown strength criterion which was proposed by the authors in 2007 and 2008 and later modified in 2013. The model involves a 3D continuous multi-segment plastic flow rule that can not only consider the effect of confining stress on the plastic flow and volumetric deformation, but also avoid the use of uncertain parameters such as dilatancy angle. The constant volume and maximum tensile stress flow rules are adopted at the high compressive confining stress condition, and the tensile stress condition, respectively. A new interpolation between the high compressive confining stress condition and the tensile stress condition is proposed to ensure the continuity of the plastic potential function over the entire stress space. The proposed constitutive model is based on the assumption that the rock mass is elastic-perfectly plastic but can be extended to the case when a rock mass shows strain-hardening behavior. The new constitutive model has been successfully implemented in a 3D finite element code, and validated by analyzing a circular ring under plane-strain loading condition and a model test tunnel with detailed instrumentation and measurements and then comparing the numerical results with the corresponding analytical solutions and experimental results. Finally, the new constitutive model is successfully applied to analyze a real highway tunnel excavated in rock mass with detailed instrumentation and measurements. The obtained tunnel roof and horizontal convergence displacements using the new constitutive model based on the modified GZZ criterion are in good agreement with those from the field measurements, further proving the importance in considering the contribution of intermediate principal stress to rock strength.

1. Introduction

Since the Hoek–Brown (HB) strength criterion was developed for intact rock by Hoek and Brown[1,2] and later extended to rock masses by Hoek and Brown[2] and Hoek et al.[4], it has been widely used in rock engineering due to the adequacy of its predictions of rock behavior and the convenience of its applications to a range of rock engineering problems. However, the HB strength criterion does not consider the effect of the intermediate principal stress although much evidence has been accumulated to indicate that the intermediate principal stress does influence the rock strength in many instances. To overcome this limitation, several three-dimensional (3D) versions of the HB strength criterion have been proposed by researchers.[5–10] Of these 3D HB strength criteria, only the criterion proposed by Zhang and Zhu[8] and Zhang[9] can predict the same strength as the original HB strength criterion at both triaxial compression and extension states and is considered a true 3D version of the original HB strength criterion. The generalized 3D HB strength criterion proposed by Zhang and Zhu was named GZZ strength criterion by Priest.[11] For simplicity, the term "GZZ strength criterion" will be used in this paper.

However, the GZZ strength criterion may have problems with some stress paths and cause inconvenience in numerical applications owing to the non-smoothness of the failure surface at both triaxial compression and extension states and the non-convexity of the failure surface at the triaxial extension state. So Zhang et al.[12] modified the GZZ strength criterion by utilizing three different Lode dependences with characteristics of both smoothness and convexity to replace its Lode dependence. The modified GZZ strength criterion not only maintains the advantages of the GZZ strength criterion, but also solves the non-smoothness and non-convexity problems with no loss of accuracy for strength predic-

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tion. Therefore, the modified GZZ strength criterion can be used as a failure criterion to construct a new constitutive model for rock masses and implemented in a 3D finite element (FE) code for 3D numerical analysis of rock engineering problems.

Several 3D numerical analysis codes including FLAC3D, PLAXIS and ZSoil are widely used in geotechnical engineering; but these codes only use the original HB strength criterion and thus virtually only 2D rock mass behavior is considered. Also, most of the 3D codes use the HB strength criterion with $a = 0.5$ and thus cannot consider rock masses with $a \neq 0.5$ frequently encountered in practice, where a is a constant reflecting the characteristics of rock masses as described in detail in next section. Therefore, in this paper, a new constitutive model based on the modified GZZ strength criterion is proposed for truly 3D analysis of rock engineering problems. The new constitutive model involves a 3D continuous multi-segment plastic flow rule and has been successfully implemented in the 3D FE code GeoFBA3D. The implemented model is verified by analyzing a circular ring under plane-strain loading condition and a model test tunnel with detailed instrumentation and measurements and then comparing the numerical results with the corresponding analytical solutions and experimental results. Finally, the implemented model is successfully used to analyze a real highway tunnel excavated in rock mass.

2. Modified generalized 3D HB strength criterion

This section briefly describes the modified generalized 3D HB strength criterion (i.e. the modified GZZ strength criterion) in order to provide the required background for the development of the new constitutive model in next section.

For intact rock, the original HB strength criterion[1,2] may be expressed in the following form

$$\sigma_1 = \sigma_3 + \sigma_c\left(m_i\frac{\sigma_3}{\sigma_c} + 1\right)^{0.5} \tag{1}$$

where σ_c is the unconfined compressive strength of the intact rock; σ_1 and σ_3 are respectively the major and minor effective principal stresses; and m_i is a material constant for the intact rock, which depends upon the rock type (texture and mineralogy) and can be obtained from the empirical value table.[13,14]

For jointed rock masses, the generalized form of the HB criterion,[3,4] which incorporates both the original and the modified forms, is given by

$$\sigma_1 = \sigma_3 + \sigma_c\left(m_b\frac{\sigma_3}{\sigma_c} + s\right)^{a} \tag{2}$$

where m_b is the material constant for the rock masses; and s and a are constants that depend on the characteristics of the rock masses.

Hoek et al.[15] proposed relationships between m_b, s, a, and Geological Strength Index (GSI) by introducing a new parameter D which is a factor that depends on the degree of disturbance due to blast damage and stress relaxation. The values of D range from 0 for undisturbed in situ rock masses to 1 for very disturbed rock masses.

$$m_b = \exp\left(\frac{\mathrm{GSD} - 100}{28 - 17D}\right)m_i \tag{3a}$$

$$s = \exp\left(\frac{\mathrm{GSD} - 100}{9 - 3D}\right) \tag{3b}$$

$$a = 0.5 + \frac{1}{6}[\exp(-\mathrm{GSI}/15) - \exp(-20/3)] \tag{3c}$$

In order to take account of the effect of the intermediate principal stress, Zhang and Zhu[8] proposed a 3D version of the HB strength criterion for intact rock and rock mass with $a = 0.5$ by combining the general Mogi[16] criterion and the HB strength criterion. Later, Zhang[9] extended the 3D HB strength criterion to the general case with any a value, which is expressed as

$$\frac{1}{\sigma_c^{(1/a-1)}}\left(\frac{3}{\sqrt{2}}\tau_{oct}\right)^{1/a} + \frac{m_b}{2}\left(\frac{3}{\sqrt{2}}\tau_{oct}\right) - m_b\sigma_{m,2} = s\sigma_c \tag{4}$$

where τ_{oct} is the octahedral shear stress and $\sigma_{m,2}$ is the mean stress defined respectively by

$$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2},\ \sigma_{m,2} = \frac{\sigma_1 + \sigma_3}{2} \tag{5}$$

in which σ_1, σ_2 and σ_3 are, respectively, the major, intermediate and minor effective principal stresses.

The criterion proposed by Zhang and Zhu [8] and Zhang [9] was named GZZ strength criterion by Priest[11] and recommended as an International Society for Rock Mechanics (ISRM) suggested method in "Three-dimensional failure criteria based on the HB criterion".[11]

To address the non-smoothness and non-convexity problems for the GZZ strength criterion, Zhang et al.[12] used three Lode dependences with smoothness and convexity characteristics, the elliptical dependence,[17] the hyperbolic dependence[18] and the spatial mobilized plane dependence,[19] to replace the Lode dependence of the GZZ strength criterion. According to the evaluation of the modified GZZ criteria,[12,20,21] the criterion modified using the hyperbolic dependence or the spatial mobilized plane dependence has relatively better prediction accuracy than that using the elliptical dependence for both intact rocks and jointed rock masses. Moreover, the criterion modified using the spatial mobilized plane dependence may cause some difficulty in judging the stress state when the Lode angle θ_σ is near 0. Therefore, the modified GZZ criterion with the hyperbolic dependence will be used for the development of the constitutive model in this paper and is presented below.

The modified GZZ strength criteria with the hyperbolic dependence can be expressed as

$$\sqrt{J_2} = L(\theta_\sigma)_H\sqrt{J_{\max}} \tag{6}$$

where J_2 is the second deviatoric stress invariant and defined by $J_2 = [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]/6$; the subscript H stands for the hyperbolic dependence which is expressed as[18]

$$L(\theta_\sigma)_H = \frac{2\delta(1-\delta^2)\cos(\pi/6-\theta_\sigma) + \delta(\delta-2)\sqrt{4(\delta^2-1)\cos^2(\pi/6-\theta_\sigma) + (5-4\delta)}}{4(1-\delta^2)\cos^2(\pi/6-\theta_\sigma) - (\delta-2)^2} \tag{7}$$

in which, the aspect ratio δ and the Lode angle θ_σ are defined as

$$\delta = \sqrt{J_{\min}/J_{\max}} = \sqrt{J_2(-\pi/6)/J_2(\pi/6)},\ \theta_\sigma = \tan^{-1}\left[\frac{\sigma_1 + \sigma_3 - 2\sigma_2}{\sqrt{3}(\sigma_1 - \sigma_3)}\right] \tag{8}$$

where $J_{\max}$ and $J_{\min}$ are the maximum and minimum values of J_2 on the same deviatoric stress plane [i.e. at the same first stress invariant ($I_1 = \sigma_1 + \sigma_2 + \sigma_3$)], respectively. $J_{\max}$ is equal to J_2 when $\theta_\sigma = \pi/6$ at triaxial compression state, and $J_{\min}$ is equal to J_2 when $\theta_\sigma = -\pi/6$ at triaxial extension state.

For a rock mass with $a = 0.5$, the explicit expression of $J_{\max}$ and $J_{\min}$ can be derived as

$$\sqrt{J_{\max}} = \frac{1}{6\sqrt{3}}\left(\sqrt{m_b^2\sigma_c^2 + 12m_b\sigma_c I_1 + 36s\sigma_c^2} - m_b\sigma_c\right) \tag{9}$$

$$\sqrt{J_{\min}} = \frac{1}{3\sqrt{3}}\left(\sqrt{m_b^2\sigma_c^2 + 3m_b\sigma_c I_1 + 9s\sigma_c^2} - m_b\sigma_c\right) \tag{10}$$

Then Eqs. (9) and (10) can be used to calculate the aspect ratio with Eq. (8) and then Eq. (6) can be directly used to determine the strength with given σ_2 and σ_3. For a rock mass with $a \neq 0.5$, the numerical solution procedure below can be used[12]: