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Closed-form approximations to borehole stresses for weak transverse isotropic elastic media

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ABSTRACT

We have derived new closed-form approximations to borehole stresses for weak tilted-transverse-isotropic media with boreholes oriented in the plane of isotropy but not necessarily along a principal stress direction. By introducing Green's three anisotropic parameters, ω_1, ω_2 and ω_3, into Lekhnitskii's formalism for anisotropic elastic media to calculate stresses around boreholes subjected to internal pressure and in situ stress, we first have recast the general borehole stresses into more compact expressions that are useful to highlight explicitly the impact of the elastic anisotropy. We have shown the equivalence between Lekhnitskii's and Green's equations for the in-plane components and extended Green's expressions to the anti-plane problem. By linearizing the expressions for weak anisotropy degree, i.e. $|\omega_i| \ll 1$, we have shown that the borehole stress expressions are the sum of the isotropic "Kirsch" solution and additional terms that depend on the anisotropic parameters $\omega_1 + \omega_2$ and ω_3. The additional terms have azimuthal/radial functional dependence of higher orders $(\cos 4\theta, \sin 4\theta)$ and $(1/r^2, 1/r^4, 1/r^6)$ for in-plane components, and, $(\cos 3\theta, \sin 3\theta)$ and $(1/r^2, 1/r^4)$ for anti-plane components. Our verification examples show that the new borehole stress expressions provide good approximations not only for weak anisotropy but also moderate and strong anisotropy in certain cases.

1. Introduction

The presence of a borehole in a stressed subsurface rock formation alters the local principal stress directions and magnitudes around the borehole and away from it over a distance of several borehole radii. For isotropic elastic homogeneous rocks (hereafter called ISO), borehole stresses are given by the classical Kirsch elastic solution[1] or its generalized version for nonaligned borehole and stress directions.[2,3] Borehole stresses depend on the far-field in situ stress, the orientation of the borehole with respect to the principal stress directions, the wellbore pressure and the material Poisson's ratio. These solutions are very convenient for practical purposes as all borehole stress components are independent of the material elastic properties, but the one along the borehole axis that is only dependent on the Poisson's ratio by the virtue of the plane strain assumption. The in-plane borehole stresses in local cylindrical coordinates (i.e., σ_{rr}, $\sigma_{\theta\theta}$ and $\sigma_{r\theta}$) vary azimuthally and radially following, respectively, series of $(\cos 2\theta, \sin 2\theta)$ and of $(1, 1/r^2, 1/r^4)$, whereas the anti-plane stresses (i.e. σ_{rz} and $\sigma_{\theta z}$) vary azimuthally and radially following, respectively, series of $(\cos\theta, \sin\theta)$ and of $(1, 1/r^2)$. However, today, most wells drilled for the purpose of natural oil and gas extraction encounter anisotropic shale formations during the drilling process either in the overburden for conventional reservoirs or in the reservoir itself for unconventional shale (mudstone) reservoirs. Most wells drilled in highly deviated or horizontal directions are penetrating transverse isotropic formations (5 independent elastic properties) or lower symmetry formations such as orthorhombic or monoclinic (9 or 13 independent elastic properties, respectively) if fracture- and stress-related effects are present. Consequently, the calculation of borehole stresses in anisotropic rocks are needed for practical applications.

The fundamentals for borehole stress and displacement analyses in anisotropic elastic media were established in two independent works starting in the 1940's: those of Green and Taylor[4–11] and those of Lekhnitskii[12,13] (originally published in Russian in 1947 and 1950, then translated into English in the 60's). Both works involve complex stress functions that were used to derive elastic solutions for holes under the generalized plane strain assumption. Amadei[14] used Lekhnitskii's formalism to calculate stresses and displacements around arbitrarily oriented open- or cased-boreholes subjected to internal pressure and in situ stress in arbitrary anisotropic elastic media (i.e. up to triclinic with 21 independent elastic properties). Those were used for practical applications by several authors.[15–21] Although Amadei's final

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mathematical expressions are closed-form, each stress component involves a series of complex analytic functions and complex coordinates that lack the simplicity of its isotropic equivalent. It is, therefore, difficult to get a physical intuition of the impact of the elastic anisotropy on the borehole stresses, i.e. how does the anisotropy change the stresses as compared to the isotropic solution (that is independent of the elastic properties) and what are the potential additional azimuthal $(\cos n\theta, \sin n\theta)$ and radial $1/r^m$ components that come into play.

By revisiting the works of Lekhnitskii,[13] we found that we can recast the general borehole stresses into more compact expressions that are useful to highlight explicitly the impact of the elastic anisotropy. The simplifications in the expressions come from the introduction of three anisotropic parameters $(\omega_1, \omega_2, \omega_3)$, following the idea of Green and Taylor,[4] into the Lekhnitskii's formalism. The three anisotropic parameters are linked to the roots of the characteristic equations following the resolution of the Beltrami-Michell equations, and are dependent on combinations of elastic properties for given material symmetries and borehole orientations. By linearizing the expressions for a weak anisotropy degree, we arrive at borehole stress expressions that are simply the sum of the isotropic solution and additional terms that depend on the elastic constants and involve an azimuthal/radial functional dependence of higher orders $(\cos 4\theta, \sin 4\theta)$ and $(1/r^2, 1/r^4, 1/r^6)$ for in-plane components, and, $(\cos 3\theta, \sin 3\theta)$ and $(1/r^2, 1/r^4)$ for anti-plane components.

The purpose of our paper is to present these new closed-form approximations to borehole stresses for weak anisotropic elastic media that highlight explicitly the impact of the anisotropy, along with the re-written more compact general expressions. In this paper, we restrict our derivations to tilted-transverse-isotropic media (hereafter called TTI) with boreholes oriented in the plane of isotropy (one example of which is a horizontal borehole in a vertical transverse isotropic medium, also called VTI medium), due to its high practical importance. This configuration leads to the decoupling of the problem into in-plane and anti-plane stress expressions. The boreholes are not necessarily along a principal stress direction, i.e. the in situ stress in the borehole reference frame may possess six non-zero components. Green [7] has published simple exact expressions for in-plane stresses at the borehole wall for this anisotropy-borehole configuration, as well as exact in-plane component solutions (i.e., σ_{rr}, $\sigma_{\theta\theta}$ and $\sigma_{r\theta}$) as function of (r, θ) for all loading conditions, but has not provided the anti-plane solutions (which are needed for a full description when the stress directions are not aligned with the borehole axis and medium symmetries). By simplifying the most general Lekhnitskii formalism for this configuration, we find the same expressions as shown by Green [7] for the in-plane problem, and also obtain the anti-plane components.

2. Borehole, stress and material anisotropy configurations

We consider an infinite elastic anisotropic formation which is homogeneous and continuous in all directions. Internally this body is bounded by a cylindrical borehole of radius a. For subsurface geologic media, we assume that the borehole is sufficiently far away from the free surface to honor the conditions of infinite media and generalized plane strain assumption.

2.1. Wellbore geometry and stress transformations

In the subsurface, an in situ stress field exists where the principal stress tensor takes the form

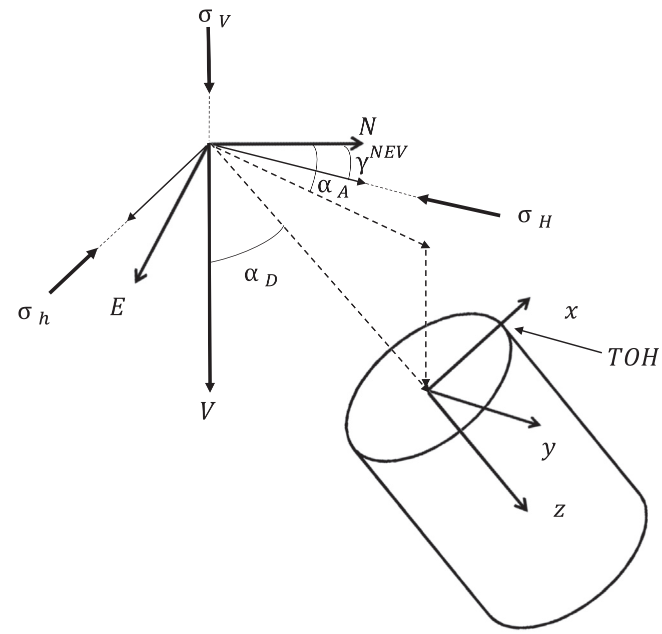

Fig. 1. Schematic of the geographic and borehole reference frames and the principal stress directions. The geographic reference frame is the north-east-vertical (NEV) frame. The borehole frame is the top-of-hole (TOH) frame whose z-axis points along the borehole in the direction of increasing depth. The x-axis is in the cross-sectional plane and points to the most upward direction, and the y-axis is found by rotating the x-axis 90^o in the cross-sectional plane in a direction dictated by the right-hand rule. The principal stress directions are chosen such as one component is parallel to the vertical NEV axis and the maximum horizontal component is rotated by the angle γ_{NEV} with respect to the north axis. The orientation of the borehole is defined by the deviation angle α_D and the azimuth angle α_A.

$$\sigma = \begin{pmatrix} \sigma_H & 0 & 0 \\ 0 & \sigma_h & 0 \\ 0 & 0 & \sigma_V \end{pmatrix} \tag{1}$$

where σ_H and σ_h are respectively the maximum and minimum horizontal principal stresses respectively and σ_V is the vertical principal stress (see Fig. 1). For the sake of simplicity, but without loss of generality, we assume that the vertical stress σ_V is always aligned with the vertical component (V) of the NEV (orthonormal right-handed North-East-Vertical) reference frame. The horizontal stress field can be rotated by an angle γ^{NEV} measured between N (north) and σ_H, towards E (east). In order to rotate the regional stress field into the NEV frame, the following coordinate transform is used

$$\sigma^{NEV} = R_z(\gamma^{NEV})\sigma R_z^T(\gamma^{NEV}) \tag{2}$$

where $R_z(\gamma^{NEV})$ is a rotation matrix defined in Appendix A and where $R_z^T(\gamma^{NEV})$ is the transpose of $R_z(\gamma^{NEV})$. For the computation of the borehole stress concentration, it is convenient to rotate the stress field into the top-of-hole borehole coordinate system, hereafter called TOH (see Fig. 1 for definition). Here and in the rest of the paper we assume for convenience that the in situ stress field is aligned with the NEV frame (i.e. $\gamma^{NEV} = 0$). The coordinate transform of the NEV stress tensor σ^{NEV} to the stress tensor in the borehole frame σ^{TOH} is

$$\sigma^{TOH} = T_t(\alpha_D, \alpha_A)\sigma^{NEV}T_t^T(\alpha_D, \alpha_A). \tag{3}$$

Here α_D and α_A are the borehole deviation and azimuth respectively. The rotation matrix $T_t(\alpha_D, \alpha_A)$ is defined in Appendix A.