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# Analytical methodology for wellbore integrity assessment considering casing-cement-formation interaction

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## ABSTRACT

A wellbore analysis presents several challenges regarding the drilling, construction and production phases. Owing to that, the cement and casing stress states during the wellbore lifespan are also difficult to predict. Nevertheless, analytical solutions can reach good results. Additionally, once analytical approaches are exhibited as mathematical expressions, they facilitate the assessment of the interaction between variables and the impact on results. On the other hand, wellbore analytical solutions do not usually consider all the phases involved. In this work, an analytical solution is developed to assess the stresses during the drilling, construction and production phases. Three models are adopted to be compared with a numerical approach. Thus, single and double casing configurations are analyzed. In both approaches, the initial stress states in the formation and in the cement, as well as the drilling operation, the casing run, the pressure changes, and the pore pressure variations are considered. The solutions consider plane strain conditions, continuous homogeneous isotropic media and linear elastic materials. In addition to the casing integrity, radial and tangential stresses for casing, cement and formation are presented. In conclusion, this analytical solution reaches excellent results when compared to the numerical model, therefore, becoming an alternative for the stress evaluation for everyday scenarios. Capable of simulating different scenarios regarding the wellbore stresses and integrity, this analytical solution is a good tool for a preliminary design.

## 1. Introduction

The challenging production scenarios of wellbore operations require that different issues be investigated to improve their reliability. Willson[16] presents a literature review of blowout occurrences. This is one of the numerous types of failures that can happen during the wellbore operation lifetime. Henderson & Hainsworth[9] chose a specific well (Elgin G4) to highlight its hazards, the lessons learned and the measures to prevent recurrence of an incident that released hydrocarbons to the atmosphere. Gasda et al.[7] believe that the most important human activity regarding leakage potential is well drilling. Chen et al.[4] present the factors affecting wellbore stability, such as the *in situ* stress field, the properties of rocks and the pore pressure of the formation.

Many studies regarding analytical solutions have been carried out considering different phenomena. Honglin et al.[10] present a solution considering the casing and the cement surrounded by the formation. The thick walled cylinder theory is used for all three materials. The solution considers internal pressure acting inside the casing. In addition, for the contact between the casing and the cement, a fully sticky condition is adopted. The same condition holds for the cement and the formation. Instantaneous uniform temperature increment is applied to all materials. The effects of the drilling operation, the initial stress states in the formation and in the cement, and pore pressure variations are not investigated.

Teodoriu et al.[15] also present an analytical solution for the same model as the one by Honglin et al.[10]. The thin walled cylinder theory is used for the casing, and the thick walled cylinder theory is used for the cement and the formation in Teodoriu et al. [15]. Different values of temperature variation are considered for each material.

More sophisticated solutions have been developed with additional parameters and more complex assumptions. Shahri et al.[14] include thermo-pore-elastic coupling by the simulation of the drilling operation. The effects of temperature and pore pressure changes around the wellbore are incorporated in their solution. On account of these parameters, the increase of the fracture re-initiation pressure, known as “wellbore strengthening”, is analyzed. Atkinson & Eftaxiopoulos[1] also study the effect of fracture in the formation, but disregard the

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underlying effects.

The objective of the current study is to develop an analytical solution that represents the stresses in the wellbore. The main steps of the wellbore life are considered: initial stress state in the formation, drilling operation, casing construction, initial stress state in the cement, pressure changes inside the casing and pore pressure variation due to production and injection. A thorough description of the well construction model can be found in Mackay & Fontoura[11] and Gray et al.[8].

The solution considers plane strain conditions, homogeneous medium and linear elastic materials. Radial and tangential stresses are assessed in casing, cement and formation. The analytical solution is compared to results of pore elastic stress analyses under drained conditions using the Finite Element (FE) method.

## 2. Analytical model development

The development of the analytical model follows the steps of the wellbore lifespan: drilling, construction and production. In a simple manner, the drilling process uses a drill bit to make a cylindrical hole in the rock. After that, the casing is positioned inside the well. As part of the construction process, the cement slurry is deployed into the well and undergoes a hardening process. If a second casing is required, its positioning is done in the construction step as well. These operations occur in a short period of time when compared with the production phase, when both the internal pressure of the casing and the pore pressure of the formation can change, resulting in stress state variations. In the development that follows each step of the drilling, construction and production phases will be represented by a specific number: (0) initial stresses (before construction), (1) outer casing and cement installation, (2) production for single casing, (3) a double casing design (if considered), and (4) production for double casing.

### 2.1. Assumptions

The solution includes the effects of initial stresses, drilling operation, casing construction and cementing, and pore pressure variation. After the cement hardening, its initial stress state is equal to the hydrostatic stress produced by the slurry weight. The solution considers plane strain condition, continuous homogeneous isotropic media and linear elastic materials. The combined cylinder model of double casing is shown in Fig. 1.

The radii are indicated in Fig. 1, where $a$ is the inner radius of the production casing, $b$ is the distance from the center of the wellbore to the interface between the production casing and the inner cement, $c$ is the distance to the interface between the inner cement and the outer casing, $d$ is the distance to the interface between the outer casing and the outer cement, $e$ is the last interface distance from the center of the wellbore, and $f$ is the distance to the end of the formation. The dimension $f$ defines the model size and should be set to a large value, for example 100 times the well radius, to achieve the correct stress state in the formation at the model boundary.

### 2.2. Single casing analytical model

#### 2.2.1. Drilling

The first step includes rock excavation and injection of the drilling fluid. The internal pressure $p_{drill}$ represents the drilling fluid pressure and $\sigma_h^{(0)}$ the total initial stress state in the formation. The Bradley equations give a good representation of the stresses in this situation. These stresses are expressed in terms of polar coordinates $r$ and $\theta$, where $r$ represents the distance from the borehole axis, and $\theta$ is the azimuth angle relative to the $x$-axis. According to Bradley[3] apud Fjær et al.[5] radial and tangential stresses are given by Eqs. (1) and (2), respectively.

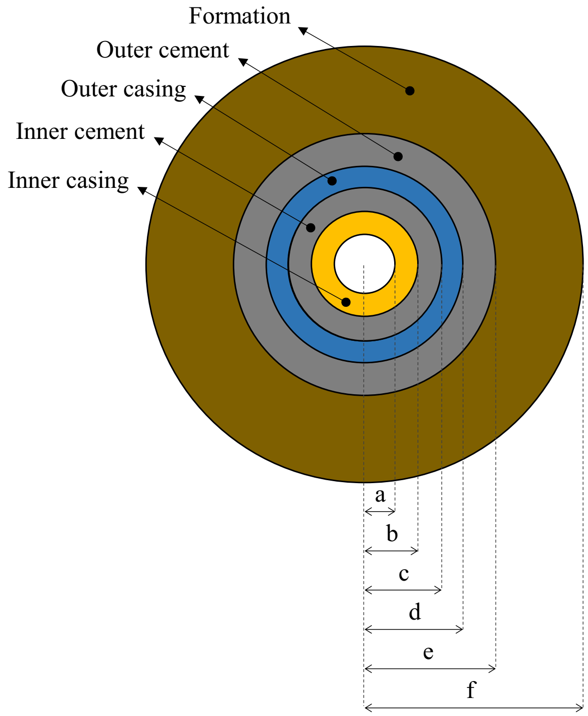

**Fig. 1.** Sketch of the combined cylinder model of casing-cement sheath-formation.

$$\sigma_{rF}^{(0)} = \left(\frac{\sigma_x^{(0)}+\sigma_y^{(0)}}{2}\right)\left(1-\frac{e^2}{r^2}\right) + \left(\frac{\sigma_x^{(0)}-\sigma_y^{(0)}}{2}\right)\left(1+\frac{3e^4}{r^4}-\frac{4e^2}{r^2}\right)cos(2\theta) + \tau_{xy}^{(0)}\left(1+\frac{3e^4}{r^4}-\frac{4e^2}{r^2}\right)sin(2\theta) + p\cdot\frac{e^2}{r^2} \tag{1}$$

$$\sigma_{\theta F}^{(0)} = \left(\frac{\sigma_x^{(0)}+\sigma_y^{(0)}}{2}\right)\left(1+\frac{e^2}{r^2}\right) - \left(\frac{\sigma_x^{(0)}-\sigma_y^{(0)}}{2}\right)\left(1+\frac{3e^4}{r^4}\right)\cos(2\theta) - \tau_{xy}^{(0)}\left(1+\frac{3e^4}{r^4}\right)\sin(2\theta) - p\cdot\frac{e^2}{r^2} \tag{2}$$

In Eqs. (1) and (2), $\sigma_{rF}^{(0)}$ and $\sigma_{\theta F}^{(0)}$ represent the radial and tangential stresses of the formation after the drilling operation, $e$ is the radius of the hole and $p$ is the inner pressure.

This analysis is carried out locally for a specific depth using a cross section of the wellbore. Assuming the shear stress $\tau_{xy}^{(0)}$ is equal to zero and isotropy of the horizontal stresses, $\sigma_x^{(0)}$ and $\sigma_y^{(0)}$ have the same value of the total initial stress $\sigma_h^{(0)}$, Eqs. (3) and (4). This assumption has some limitations when applied to real situations. However, it is practical and, for some cases, appropriate. The sign convention used throughout the paper is positive for tension and negative for compression.

$$\sigma_{rF}^{(0)} = (\sigma_h^{(0)} + pp)\left(1 - \frac{e^2}{r^2}\right) + (-p_{drill} + pp)\cdot\frac{e^2}{r^2} \tag{3}$$

$$\sigma_{\theta F}^{(0)} = (\sigma_h^{(0)} + pp)\left(1 + \frac{e^2}{r^2}\right) - (-p_{drill} + pp)\cdot\frac{e^2}{r^2} \tag{4}$$

Here, $pp$ is the formation pore pressure while expression $(\sigma_h^{(0)} + pp)$ is the formation effective stress and $(-p_{drill} + pp)$ is the pressure acting inside the borehole [12].

As it can be seen, after the drilling operation, the solutions for effective stresses $\sigma_{rF}^{(0)}$ and $\sigma_{\theta F}^{(0)}$ are axisymmetric.

#### 2.2.2. Casing installation

In this step, it is assumed that casing and cement are set simultaneously. At this point, the initial stresses in the cement are