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Fractal properties of joint roughness coefficients

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1. Introduction

Rock joints represent discontinuities in rock masses, and are sources of mechanical instabilities of rock structures. The larger asperity roughness of joint surfaces contributes to a better stability of these structures, and thus assessment of joint roughness is an important part of geotechnical practice when shear strength of rock joints is evaluated.

Barton[1] introduced the concept of joint roughness coefficients (*JRC*) to characterize surface roughness of unfilled rock joints. The same author, in co-operation with Choubey,[1,2] showed experimentally that shear strength of rock joints depends strongly on the values of *JRC*, and proposed the following relation for determining shear strength τ:

$$\tau = \sigma_n \, tg\left[JRC\cdot \log\left(\frac{JCS}{\sigma_n}\right) + \Phi_b\right] \tag{1}$$

where σ_n is the normal effective stress, *JCS* represents effective joint wall compressive strength, and Φ_b is a basic friction angle (material constant). The difficulty of evaluating τ consists in the difficulty of determining *JRC*. Despite this fact, *JRC* values are the most frequently used characteristics for quantifying joint roughness and shear strength of rock joints.

Barton and Choubey[1,2] also proposed to estimate *JRC* values by means of three possible ways: (i) by back-computing from relation (1), (ii) by using tilt and pull tests or (iii) by a simple *visual comparison* of the analyzed joint surfaces with ten standard two-dimensional (2D) profiles of known *JRC* values. The *visual assessment* of joint roughness due to its simplicity and quick accomplishment often is selected as a preferable method, although it is based on rather a subjective procedure.

Besides the joint roughness coefficients associated with Barton's visual method, there are various other roughness coefficients that may characterize surface irregularities.[3–8] However, the joint roughness coefficients are very special indicators derived from experimental measurements of shear strength,[1] which makes them well suited for geotechnical practice.

Soon after Barton's *JRC* concept had been published,[1] many papers appeared[9–17] that tried to express the coefficients *JRC* in various functional forms. For example, Lee et al.[13] proposed the joint roughness coefficient *JRC* in the form of quadratic polynomial, in which fractal dimension assumed the role of independent variable. These authors computed the fractal dimension *D* from two-dimensional profiles by the yard-stick method, and suggested a regression equation as follows:

$$JRC = -0.87804 + 37.7844\left(\frac{D-1}{0.015}\right) - 16.9304\left(\frac{D-1}{0.015}\right)^2 \tag{2}$$

An instructive overview of various methods utilizing joint roughness coefficients in current geotechnical practice has recently been published by Morelli[18]. A comprehensive overview of existing empirical fitting patterns for *JRC* can be found in two other recently published papers.[19,20] The first paper by Li and Zhang[19] summarizes those patterns which employ various measurable topographic parameters. The second paper by Li and Huang[20] summarizes existing empirical fitting patterns for *JRC* that are based on the *fractal dimensions D* of the measured profiles of jointed surfaces. They gathered nineteen empirical fractal patterns that can be divided into five groups:

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$$JRC = \sum_{n=0}^{N} c_n D^n, \qquad N = 1 \quad \text{or} \quad N = 3 \tag{3}$$

$$JRC = \sum_{n=0}^{N} c_n (D-1)^n, \qquad N = 2 \tag{4}$$

$$JRC = c\,(D-1) \tag{5}$$

$$JRC = c\,(D-1)^{\beta} \tag{6}$$

$$JRC = f\,(D-1) \tag{7}$$

where c and β are numerical constants, and f is a function of fractional type.

All the fractal patterns in the paper by Li and Huang[20] represent *empirical relations*. Their coefficients c, c_n, β and others that are hidden in the fractional functions associated with group (7) were optimized by regression methods. Their D-domains were restricted to very small intervals $D \in (1\ ,\ 1.09)$. These very low D values were mostly derived from the ten Barton standard profiles by using the divider method, sometimes called the compass-walking method.[20] However, it is not likely that natural rock joints generally have such small fractal dimensions. The values about $D \approx 1.09$ of *profile curves* represent very smooth surfaces. In practice, when analyzing natural rock joints, some researchers have found larger D values. For example, when analyzing rock surfaces at Yucca Mountain in Nevada, Car[21] found power spectrum fractal dimensions reaching up to 1.467, whereas divider (compass-walking) fractal dimensions for the same surfaces did not exceed the value 1.032. As will be explained in the next sections, this is because the profile curves of rock surfaces are not self-similar fractals, but rather self-affine fractals, which require a *modified divider method* to obtain rigorous fractal dimensions.

Finally, another important point of the paper by Li and Huang[20] should be highlighted. These authors correctly identified three main sources of numerical discrepancies between the studied fractal fitting patterns, namely, (i) different origins of joint profiles, (ii) different numerical methods for computing D, and (iii) different methods for determining JRC. In the next sections, a new fractal fitting pattern will be derived that does not suffer from these sources of drawbacks, since the derivation employs only those scaling relations that are generally valid in the field of fractal geometry. The derivation will neither compute any fractal dimensions, nor use profiles of other authors, so that no regression procedure will be used, and even the employed values of JRC will be received from the basic relations introduced by their inventors, i.e. by Barton[22,23] and Bandis.[24,25]

2. Size effect with joint roughness coefficients

JRC values are correlated with the maximum profile asperity amplitude $a_{\max}$ measured along the profile length L. This fact was found by Barton[22] on the basis of the results of Bandis.[24] Barton[22] suggested the following two relationships:

$$JRC_o = 400\ \frac{a_{\max}}{L_o} \quad \text{for} \quad L_o = 10\ \text{cm} \tag{8}$$

$$JRC = 450\ \frac{a_{\max}}{L} \quad \text{for} \quad L = 1\ \text{m} \tag{9}$$

and accompanied them by a figure illustrating experimental measurements of the mentioned correlations. In addition, Barton and Bandis[23,25] published complementary relations concerning JRC, JRC_o, JCS and JCS_o:

$$JRC = JRC_o \left(\frac{L}{L_o}\right)^{-0.02 \cdot JRC_o} \tag{10}$$

$$JCS = JCS_o \left(\frac{L}{L_o}\right)^{-0.03 \cdot JRC_o} \tag{11}$$

Relationships (8)–(11) strongly suggest the scaling relations frequently appearing in fractal geometry.[26–33] They will be subjected to analysis and their functional forms will serve as prototypes for deriving the rigorous fractal scaling formulae.

3. Fractal scaling relations

When computing fractal dimension of a geometrical object, it is necessary to decide whether the object is self-similar or self-affine. The fractal dimensions of self-similar objects can be computed by all known fractal methods as box-counting method,[26] compass method,[26–28] 'mass' method,[26] power spectral method[27,28] or methods using residuals as e.g. R/S method,[26,28] variance (root-mean-square) method,[29] range method,[30,31] etc. However, the computation of fractal dimensions of self-affine fractals requires a careful choice of modified computational methods.[32,33] The problem is that self-affine curves, in contrast to self-similar ones, are not identically scaled in x- and y-directions, and the common computational methods have to be adapted to this self-affine property.[27,29–31] The different scaling in the x- and y-directions[32] means that when the x-coordinates of a self-affine curve are multiplied by a positive constant b, then the corresponding y-variables have to be multiplied by a *different* positive constant b^H in order to receive statistically similar curves, i.e.,

$$y(bx) \to b^H y(x), \qquad 0 \le H \le 1, \tag{12}$$

where H is the so-called Hurst exponent,[27–32] and b is the so-called scaling constant. Relation (12) manifests the *statistical identity* of the curves $y(x)$ and $b^{-H} y(bx)$. The Hurst exponent H of two-dimensional self-affine curves is closely connected with the fractal dimension D of $y(x)$ as follows[27–32]:

$$D = 2 - H \tag{13}$$

Since the two-dimensional *vertical* profiles of fracture surfaces are self-affine curves (self-affine fractals),[26–28,32] their dimensions D have to be computed by proper methods. For example, the residual methods, namely the *range method*,[30,31] are quite convenient for such a task. The range method is based on the following scaling formula[30,31] that is generally valid for self-affine fractals:

$$a_{\max} = const\ \times L^H \tag{14}$$

where $a_{\max}$ is the y-range of the self-affine function $y(x)$, i.e. $a_{\max} = |MAX(y) - MIN(y)|$ within the L-domain, i.e. $x \in (0,\ L)$. As seen, the symbols $a_{\max}$ and L in Eq. (14) have identical meanings as the symbols in Barton's Eqs. (8) and (9). To illustrate the applicability of the scaling relation (14) to rock joints, we plot the functional dependence $a_{\max}(L)$ in Fig. 1. This figure has been formed on the basis of graphical data published originally by Barton.[22] Fig. 1 uses the log-log co-ordinate system in which the graph of the function $a_{\max}(L)$ shows linear behavior

$$\log(a_{\max}) = \log(const) + H \cdot \log(L) \tag{15}$$

The Hurst exponent H appears as a slope in the log-log relation (15) and, as seen in Fig. 1, it assumes the value $H \approx 0.53$ (i.e. $D \approx 1.47$). The graph has been optimized by the least square method. The Hurst exponent 0.53 satisfies the requirement $0 \le H \le 1$. The experimental data used for plotting the linear dependence (15) span over almost two orders of joint profile lengths L (starting from 10 cm up to several meters), which instructively illustrates the universal applicability of fractal relation (14) to the joint profiles of wide length scales.

On the basis of the general scaling relation (14), it is possible to rewrite Barton's Eqs. (8) and (9) into the following scaling forms:

$$JRC_o = 400\ \times const \times L_o^{H-1} \tag{16}$$

$$JRC = 450\ \times const \times L^{H-1} \tag{17}$$

From Eqs. (16) and (17), a more compact fractal formula can be derived: