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A modified method for predicting the stresses around producing boreholes in an isotropic in-situ stress field

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ABSTRACT

Rock formations are always under in situ stresses due to overburden or tectonic stresses. Drilling a well will lead to stress redistribution around the well. Understanding such a stress redistribution, and adopting a proper failure criterion, play a vital role in predicting any potential wellbore failure. However, most of the published analytical models are based on assumptions that do not satisfy the boundary conditions during production, that is, when the well pressure is less than the pore pressure. This paper is aimed at the modeling of the stress regime around the wellbore through combining the poroelastic model with proper boundary conditions under different flow regimes. As a result, a new model is presented to calculate the stresses around the well under Darcy and non-Darcy flow conditions. Covering the non-Darcy regime makes the new model applicable to gas reservoirs with non-Darcy flow. The results of this new model have been compared with the results of other models previously published in the literature. The results show that the new model convergence at the boundary condition is superior to other models used in the oil and gas industry.

1. Introduction

Understanding the in-situ stress distributions around oil and gas wells is one of the major factors in geomechanical modeling of reservoirs. By determining the magnitude and orientation of stresses and pore pressure around the wellbore, one will be capable to resolve wellbore instability problems during drilling, production and injection.[2,3] In general, each rock mass in the fields is subjected to natural stresses because of the overburden mass weight and tectonic forces.[4] Drilling a well will lead to stress redistribution due to removal of some reservoir rock portions. This causes stress concentration on the wellbore wall and a new redistribution of stresses in the vicinity of the wellbore[5] to take place (Fig. 1).

Stress in rocks is usually presented as a tensor with nine components. However, only six of such components are independent.[6] The study of stress tensor in a rock mass can be simplified by studying three principal stresses, that is, the vertical stress (σ_v), the maximum horizontal stress (σ_H) and the minimum horizontal stress (σ_h), and their orientations.[3] In addition to the three principal stresses and their orientations, the formation pore pressure should be considered.[7] In practice, to overcome the pressure loss due to formation damage, higher pressure drawdowns are commonly applied in the wells. Such a strategy will increase the hydrocarbon production rate, and will improve the recovery factor. However, if the rock strength during the applications is exceeded, then sand production from the wellbore will be faced.[5] As the well pressure is playing a significant role in wellbore stability, the designed well applications should be adjusted to ensure the generation of acceptable stress concentrations around the well, that maintain stability.

Borehole stability of oil and gas fields, requires a good knowledge about the existing in situ stresses, the elastic and plastic properties of the rock, formation pore pressure, inclination and azimuth of well, and a proper selection of a shear failure criterion. For the task of designing the optimum drilling trajectory with respect to mechanical wellbore stability, the optimum drilling trajectories for wells are strongly depending on the orientations and magnitudes of the in situ stresses.[8,9] In addition to those aforementioned parameters, the formation temperature has received less attention although it has a significant impact on the in situ stresses. By increasing the formation temperature, the rocks will go under compressive conditions which then brings an increase in principle stresses near the well.[10] In this regard, Yan et al.[10] modeled the temperature distribution at the bottom of a hole and used it to calculate the stress distribution around the well in high temperature formations.

In comparison with linear elastic modeling of rock failure, elasto-plastic modeling is considered generally more realistic.[11] Bradford and

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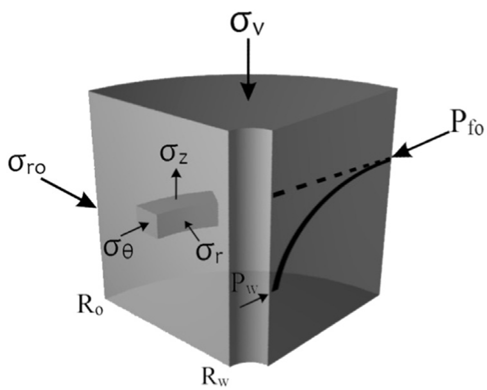

Fig. 1. Schematic of hollow cylinder model.

Cook[12] presented an elasto-plastic model to predict the wellbore instability in vertical boreholes that considers isotropic in situ stress and the pore pressure of fields under unsteady state flow regime conditions. In their study, the pore pressure was obtained by linear diffusion equation. Sanfilippo et al.[13] assumed that the pressure in the drainage area of a well is uniform and introduced a poro-elasto-plastic model for the reservoir with infinite acting boundary conditions. Risnes et al.[1] studied sand stresses around a wellbore by considering the steady state flow conditions for an incompressible fluid in an isotropic in situ stress field. They used elasticity and plasticity theories and concluded that there is a plastic zone near the wall of wellbore because of the generated stress concentration. They found that the radius of the plastic zone decreases as rock becomes more consolidated. They showed that the pore pressure increment causes the plastic region reduction around the well. They also demonstrated that the near wellbore permeability plays a key role in the stability criterion. In practice, the analytical determination of plastic deformation before the failure is generally difficult.[11,14] Therefore most researchers have used linear elastic model in their study to analyze the wellbore stability,[8,15–18] as it is more practical to apply.

Besides the analytical modeling of wellbore stability, the numerical modeling methodologies have received a lot of attention. For instance McLean and Addis[11] compared numerical results with that of a linear elastic model in a horizontal well drilled in the North Sea. They showed that their numerical model predicts the stresses more accurately in comparison with elastic analysis. Other researchers used coupled numerical methods. Such methods are based on coupling of thermo-chemo-poro-elasto-plastic models to obtain pore pressure and stress changes near well in chemically active formations.[19] The common negative issues with such methods are the high computation times and the required high accuracy of the input parameters.[11]

In this work, at first a comprehensive review of studies about linear poroelastic models that consider varying and constant pore pressure are covered. Then, a modified model that improves the previously applied models at operational boundary conditions is proposed. After that, a new set of equations is introduced to determine the stress magnitudes around the wellbore during non-Darcy radial flow. This model is developed such that it can be applied to both compressible and incompressible fluids under high flow rate and steady state conditions.

2. Model description

In order to determine the stresses around a borehole, a hollow cylinder model is used and expressed in a cylindrical coordinate system. In such a model, well can be assumed as an infinite vertical hole with full axial symmetry, inner radius as R_w, pressure at the wall as P_w, outer radius of the reservoir as R_o, and pressure at the reservoir boundary as σ_{ro}. It is assumed that the cylinder is loaded by a vertical stress σ_v. Under isotropic condition and for a homogenous rock, it is assumed that stresses and strains are independent of height and rotation angle. By neglecting body forces, the equilibrium equation can be written as.[6]

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0, \tag{1}$$

Where σ_r is the radial stress, σ_θ is tangential stress, and r represents the radius.

In addition, through using Hooke's law in cylindrical coordinates, the effective radial, tangential and vertical stresses can be expressed as

$$\sigma_r = (\lambda_{fr} + 2G_{fr})\varepsilon_r + \lambda_{fr}\varepsilon_\theta + \lambda_{fr}\varepsilon_z + \alpha P_f, \tag{2}$$

$$\sigma_\theta = \lambda_{fr}\varepsilon_r + (\lambda_{fr} + 2G_{fr})\varepsilon_\theta + \lambda_{fr}\varepsilon_z + \alpha P_f, \tag{3}$$

$$\sigma_z = \lambda_{fr}\varepsilon_r + \lambda_{fr}\varepsilon_\theta + (\lambda_{fr} + 2G_{fr})\varepsilon_z + \alpha P_f, \tag{4}$$

where P_f is the formation pore pressure, λ and G are Lamé's parameters, α is the Biot coefficient, and ε_r, ε_θ and ε_z are the strains in the r, θ, and z directions, respectively.

Strains can be related to the displacements in cylindrical coordinates (u, v, and w) and defined by

$$\varepsilon_r = \frac{\partial u}{\partial r}, \tag{5}$$

$$\varepsilon_\theta = \frac{u}{r} + \frac{1}{r}\frac{\partial u}{\partial \theta}, \tag{6}$$

$$\varepsilon_z = \frac{\partial w}{\partial z}. \tag{7}$$

Since the deformation is only taking place in the radial direction, the axial strain will become zero and the partial differential term in the tangential strain will be equal to zero too (i.e., $\frac{1}{r}\frac{\partial u}{\partial \theta}$=0). After inserting Eqs. (5) and (6) into Eq. (2) (*Hook's* law), Eq. (1) can be expressed by

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \frac{u}{r^2} + \frac{\alpha}{\lambda_{fr} + 2G_{fr}}\frac{dP_f}{dr} = 0, \tag{8}$$

which is a displacement equilibrium equation.

2.1. Constant pore pressure

If we assume that the pore pressure is constant, Eq. (8) can be simplified to[6]

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \frac{u}{r^2} = \frac{d}{dr}\left(\frac{1}{r}\frac{d\,(ru)}{dr}\right) = 0. \tag{9}$$

In practice, such an assumption is only valid under specific conditions. For instance, during drilling when impermeable mud cake is formed, one can assume that there are no pore pressure changes, and so the pore pressure will be assumed to stay constant in such an operation. After the integration of Eq. (9), we will have $u = C_1 r + C_2/r$. By substituting the radial displacement equation in Eqs. (5) and (6), the radial and tangential strain can be defined by $\varepsilon_r = C_1 - C_2/r^2$ and $\varepsilon_\theta = C_1 + C_2/r^2$. Thus, according to Eq. (4), σ_z will be equal to a constant value. Using Eqs. (2) and (3), the radial and tangential stresses are given by

$$\sigma_r = C_1' + \frac{C_2'}{r^2}, \tag{10}$$

$$\sigma_\theta = C_1' - \frac{C_2'}{r^2}. \tag{11}$$

To determine σ_r and σ_θ, two boundary conditions and Hooke's law should be applied. In this case, the following boundary conditions are used:

$$\sigma_r = P_w \quad for \quad (r = R_w), \tag{12}$$