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Analytical solution for assessing continuum buckling in sedimentary rock slopes based on the tangent-modulus theory

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1. Introduction

Slopes in stratified rock masses cut or naturally parallel to bedding form a non-daylighted slab structure. When these slopes contain a continuous weak bedding plane or a persisting discontinuity, and the rock mass lacks any relevant cross-jointing able to favor wedge, planar or ploughing failure, then the kinematic potential for buckling emerges. Buckling collapse hazard in a rock mass slope[1–6] suppose an absolute instability where the failure is often rapid and without previous easily recognizable warnings. When the slope reaches a critical height, any extra height, pore pressure increase or small external load could cause it to buckle. Whether in the case of a footwall slope of a surface coal mine or in a natural slope, that potential risk demands a rigorous assessment of the slope stability for proper design or mitigation. The use of the limit equilibrium technique in combination with Euler's buckling theory has resulted in some closed solutions.[7–12] However, as stated by others[1,6,13,14] and from the author's own experience, these approaches have proven unable to consistently explain the buckling failure mode. This article suggests a formula that can be used to calculate the length of the passive segment and the use of the tangent-modulus theory to identify the mode of failure and evaluate slope stability. Nine real buckling cases were analyzed with acceptable results.

2. Length of the passive segment

In a slope under buckling potential, the overall slope length, L_s, can be divided into two different regions: the "passive segment", which is at the toe of the slope, and the active region, which is the section above the passive segment and is called the "driving segment" (see Fig. 1). During failure, the "passive segment" starts bulging upward and progressively releases the "driving segment" along the dip direction in a concomitant translational failure mode.

A one span beam-column with a pin-roller support on its left end and a pin support on its right end is used to model the passive segment (see Fig. 2). The bedding parallel to the slope face is modeled as an orthogonal fracture system with a flat overall geometry. Since the beam-column is assumed to be straight, it is necessary to recreate this same natural condition in the model by introducing an auxiliary eccentric compressive force, T, at the level of the pinned supports. The magnitude of this eccentric force shall be exactly the force needed to compensate the downward deflection due to self-weight and thus keep the beam-column straight.

Beam-column cannot deflect downwards due to natural constraints, nor can they deflect upwards assuming negligible tensile strength (as is common for major surface stratified rock masses). Therefore, in this compression-only setting, the critical point from which equilibrium is lost can be defined as the moment when the beam-column begins to deflect upward. Before this critical point, the beam-column is still straight and can develop compressive strength, and after this critical point, the beam-column fails by losing its compression-only status. The word "straight" is used here to mean a beam-column in a compression-only setting; consequently, curved beam-columns while working in a compression-only setting, in essence, also adhere to this meaning.

Based on this failure mechanism, at the critical point, the length of the passive segment, ℓ , shall be such that the total midspan deflection in the beam-column model is zero. In addition, the driving force, P, is evaluated by isolating the potential driving segment as illustrated in Fig. 3. By summing the forces along dip, we can obtain the expression (1) for the resultant driving force P. In this expression, rock mass unit

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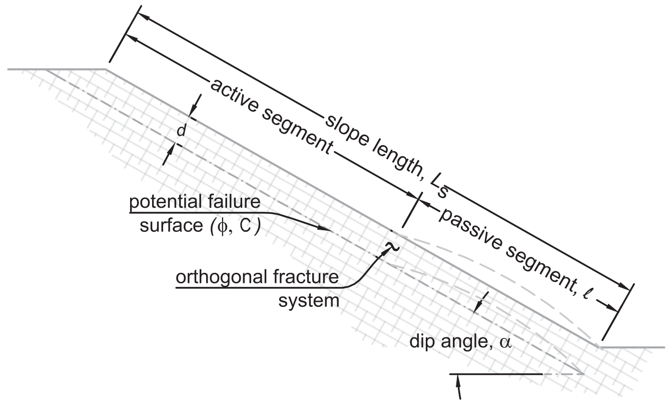

Fig. 1. Sketch of buckling failure mode in slopes of stratified rock.

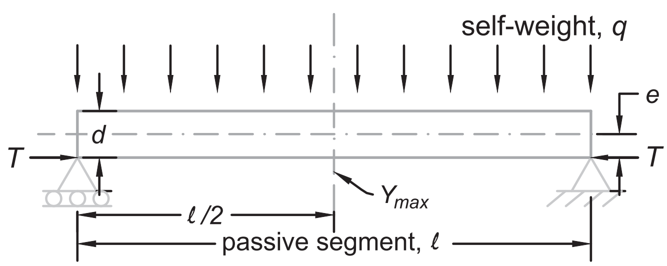

Fig. 2. Model for the passive segment (eccentrically loaded beam-column with self-weight).

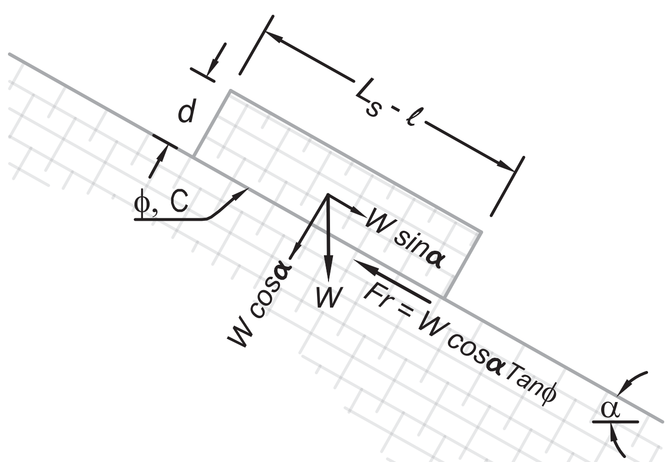

Fig. 3. Equilibrium of forces for the driving segment.

weight, γ, slab thickness, d, friction angle of the failure surface, ϕ, cohesion of the failure surface, C, slope angle, α, and dip overall slope length, L_s, are known, and the length of the passive segment along the dip, ℓ, is unknown. Under certain conditions of slope angle, depth of the potential failure surface and sliding surface friction and cohesion, it is possible to obtain a negative driving force. This negative result means that the buckling failure mode could be not a real concern in such a setting.

$$P = (\gamma d \sin\alpha - C - \gamma d \cos\alpha \tan\varphi)(L_s - \ell) \tag{1}$$

The two segments, active and passive, have been evaluated independently. To integrate both together, we need to ensure the equilibrium condition. The driving force, P, in the driving segment acts along dip direction as well as axially, assuming that the center of gravity matches the geometric center of the slab. As explained before, the eccentric force and the self-weight in the model for the passive segment cancel bending moments each other, leaving only the axial force along dip, T. Consequently, and only at the critical point, it is true that the force P shall be equal to the force T to guarantee the overall equilibrium of the slope. This equilibrium exercise implies rigid blocks; nevertheless, for the purpose of surface sliding analysis where the stress is relatively low, it is reasonable to assume that the blocks are rigid.[2]

Using deflection formulas derived from the Euler-Bernoulli beam equation[15,16] we can evaluate the midspan deflection. If only the weight of the beam is considered in the beam-column model (T=0), then the maximum downward elastic deflection ($y_{max}\downarrow$)[16 (p. 193)] is:

$$y_{max\downarrow} = 5q\ell^4/384EI \tag{2}$$

If only the eccentric force is considered in the beam-column model (q=0), then the maximum upward elastic deflection ($y_{max}\uparrow$)[16 (p. 194)] is:

$$y_{max\uparrow} = Te\ell^2/8EI \tag{3}$$

Therefore, to obtain zero midspan total deflection, through the use of the principle of superposition and setting T equal to P, we find at the critical point, the downward deflection from Eq. (2) shall be equal to the upward deflection from Eq. (3):

$$5q\ell^4/384EI = Pe\ell^2/8EI \tag{4}$$

Substituting P as defined in Eq. (1), inserting the eccentricity, e, equal to $d/2$ and then simplifying the results for Eq. (4), we obtain the quadratic equation for ℓ :

$$\ell^2 + [9.6(\gamma d \sin\alpha - C - \gamma d \cos\alpha \tan\varphi)/q]\ell - [9.6(\gamma d \sin\alpha - C - \gamma d \cos\alpha \tan\varphi)L_s/q] = 0 \tag{5}$$

The quadratic Eq. (5) has the following simplified solution for its positive root; in this equation, the constant C_1 and the beam-column self-weight, q, are known:

$$\ell = \sqrt{C_1^2 + 2C_1L_s} - C_1 \tag{6}$$

$$C_1 = 4.8(\gamma d \sin\alpha - C - \gamma d \cos\alpha \tan\varphi)(d/2)/q$$

$$q = d\gamma \cos\alpha$$

In conclusion, for any given slope, the length of the passive segment can be calculated using Eq. (6) as a function of the overall slope length along the dip. When the toe slope contains a nose or a natural imperfection, the passive segment length is no longer the one identified above. In this case, it is believed that passive segment length must be the same as the nose length or controlled by the imperfection itself.[2,3,6]

3. Compressive strength

The tangent-modulus theory by Shanley for column buckling concluded that the lateral deflection starts very near the tangent-modulus load, P_t, that is, the concept of tangential modulus predicts the maximum load that can be applied to a column before it undergoes lateral deformation.[17] Given that we previously defined the straight state as the needed requirement for the beam-column model to be in equilibrium, it is reasonable to adopt the tangent-modulus theory.

To properly investigate the buckling strength of columns loaded in the inelastic range using tangent-modulus theory, it is necessary to know the real stress-strain diagram of the rock mass uniaxial state of stress. Using previous stress-strain diagram, we can deduce the corresponding tangent-modulus, E_t. Finally, by introducing this tangent-modulus into Engesser's formula, we can obtain the buckling formulas (7) and (8) needed to assess, respectively, the compressive load and the compressive stress in the inelastic domain.[18]

$$P_t = \pi^2 E_t I/\ell^2 \tag{7}$$

$$\sigma_t = \pi^2 E_t/(\ell/r)^2 \tag{8}$$

On the other hand, the buckling force of a column in the elastic domain is obtained from Euler's buckling formulas (9) and (10) which