International Journal of Rock Mechanics & Mining Sciences 92 (2017) 19–29

Contents lists available at ScienceDirect

International Journal of Rock Mechanics & Mining Sciences

journal homepage: www.elsevier.com/locate/ijrmms

Discontinuous deformation analysis based on strain-rotation decomposition

Huo Fan[a,*], Hong Zheng[b], Jidong Zhao[a,*]

[a] Department of Civil and Environmental Engineering, The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong
[b] Key Laboratory of Urban Security and Disaster Engineering, Ministry of Education, Beijing University of Technology, Beijing 100124, China

ARTICLE INFO

Keywords:
Strain-rotation decomposition
Dynamic formulation
Discontinuous deformation analysis
Large rotation

ABSTRACT

The S-R (strain-rotation) decomposition theorem has an ability to capture strain components and rotation components at the same time. Using the principle of virtual power (VP), in this study, a new formulation independent of specific numerical methods is proposed for the analysis of dynamic large or small deformation. Then, the formulation is applied to the discontinuous deformation analysis (DDA), yielding a new DDA based on the S-R decomposition theorem, abbreviated as $SRDDA_{vp}$. Compared with the conventional DDA, $SRDDA_{vp}$ adopts a slightly modified basic variables together with the generalized-α method. The analysis of some typical examples indicates that $SRDDA_{vp}$ can naturally overcome the issue of volume expansion, effectively improve the calculation accuracy and, equip DDA with the potential to treat large deformation.

1. Introduction

The discontinuous deformation analysis (DDA) is a discrete block-based method [1,2]. In both 2D-DDA and 3D-DDA, the special shape functions and basic variables are employed to make the approximation of displacement field is independent of the shape of block. The effectiveness of DDA in geotechnical problems has been recognized [3–5], and extensively applied in the analysis of seismic landslides [6–8], crack propagations [9–11], hydraulic fractures [12,13], masonry structures [14], the path tracking of rockfalls [15], fluid-solid coupling [16] and motion of particulate media [17,18].

During the past 20 years, the performance of DDA is enhanced largely. The higher-order DDA [19], a nodal-based DDA [20], the FEM-DDA [21], the NMM-DDA [22] and the DDA with bonding springs [23] improved the deformability of objects simulated by DDA. The post-adjustment method [24], the Taylor series method [25], the trigonometric method [26], the post-contact adjustment method [27], the displacement-strain modification method [28] overcame the volume expansion of block due to small deformation assumption, and a procedure [29] to mitigate the elastic distortions with large rotation. Some convergence criterions [30], the trick of contact state recovery [31], and the strategy of strengthening the movement trend [32] speeded up the open-close iteration. The augmented Lagrange multiplier method [33], the Lagrange multiplier method [34], the complementarity method [35–37]the variational inequality method [38] improved the accuracy of contact force. The one temporary spring method [39] and the angle-based method [32] handled the indeterminacy of vertex-vertex contact. For 3D-DDA, the contact submatrices [40] modified the stiffness matrix. The models of point-to-face and edge-to-edge contact [41,42] dealt with the various contacts. An algorithm [43] coped with the frictionless vertex-to-face contacts. Another algorithm [44] searched and calculated geometrical contacts. A fast algorithm [45] identified the common plane. A multi-shell cover algorithm [46] detected contacts. A nodal-based 3D-DDA [47] was developed. Moreover, the new contact theory [48] developed by Shi is expected to significantly simplify the difficulties in treating three-dimensional singular contacts.

It is worth mentioning that the S-R decomposition theorem [49–54] is an important result in the field of geometric nonlinearity. By this theorem, the strain and local rotation can be simultaneously and accurately captured. However, a dynamic formulation based on this theorem remains absent. In this study, using the principle of virtual power (VP), a new formulation for dynamic analysis is firstly deduced. The S-R-D-based formulation is independent of specific numerical methods. In other words, it provides an opportunity to develop DDA under the background of the new theory, in which the small strain assumption is no longer needed. Compared with the conventional DDA, a slightly modified displacement function and the generalized-α[55] method are utilized in the S-R-D-based DDA, abbreviated by $SRDDA_{vp}$, in which the subscript "vp" stands for the principle of virtual power. The results obtained show that $SRDDA_{vp}$ can naturally overcome the issue of volume expansion, effectively improve the calculation accuracy and, equip DDA with the potential to treat large deformation.

* Corresponding author.
E-mail addresses: Huofan@ust.hk (H. Fan), jzhao@ust.hk (J. Zhao).

http://dx.doi.org/10.1016/j.ijrmms.2016.12.003
Received 4 March 2016; Received in revised form 17 November 2016; Accepted 7 December 2016
1365-1609/

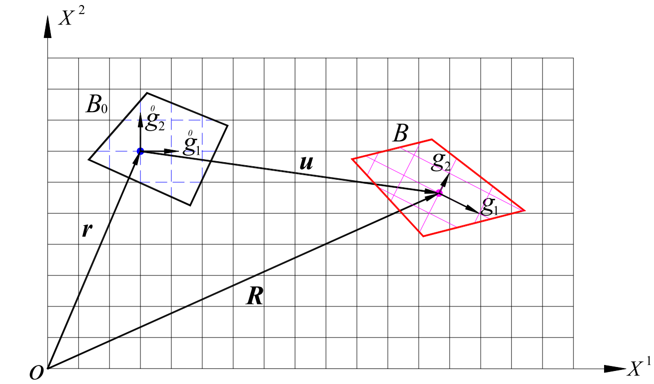

Fig. 1. Co-moving coordinate description of the motion of a deformable body.

2. S-R decomposition theorem

The S-R decomposition theorem is always associated with the co-moving coordinate description method. The connection between the theorem and the co-moving coordinate has been demonstrated and illuminated in [49–54]. Here, for completeness, we only touch upon the related concepts and theories.

For a deformable body in Euclidean space E^3, the following two reference frames are chosen to describe the motion of a body:

(1) A global reference system $\{X^i\}$ (i=1, 2, 3), which is fixed in space.
(2) A co-moving coordinate system $\{x^i\}$ (i=1, 2, 3), which is embedded in the deformable body, with its coordinate line allowed to stretch and rotate.

In general, the initial reference frame or the initial co-moving coordinate system is chosen as a rectilinear or curvilinear orthogonal system. However, owing to the occurrence of deformation of the considered body, a new curvilinear system may be formed following the deformation. Fig. 1 shows the configuration change of a co-moving coordinate system in the two-dimensional case. The situation in the three-dimensional case is similar. Let **r** and **R** be the position vectors of a point before and after deformation, and **u** the displacement vector. Then, the three vectors have the relationship

$$\mathbf{R} = \mathbf{r} + \mathbf{u}. \tag{1}$$

We define the basis vectors at a point in the initial co-moving coordinate system by

$$\overset{0}{\mathbf{g}}_i = \frac{\partial \mathbf{r}}{\partial x^i}, \quad i = 1,\ 2,\ 3. \tag{2}$$

After deformation, the basis vectors at the same point change to

$$\mathbf{g}_i = \frac{\partial \mathbf{R}}{\partial x^i}, \quad i = 1,\ 2,\ 3. \tag{3}$$

Using Eq. (1), one can obtain

$$\frac{\partial \mathbf{R}}{\partial x^i} = \frac{\partial \mathbf{r}}{\partial x^i} + \frac{\partial \mathbf{u}}{\partial x^i}. \tag{4}$$

In the curvilinear system, any vector can be decomposed with respect to the basis vector $\overset{0}{\mathbf{g}}_i$. For the displacement **u**, we have

$$\mathbf{u} = u^j \overset{0}{\mathbf{g}}_j \tag{5}$$

Further, we can obtain

$$\frac{\partial \mathbf{u}}{\partial x^i} = \frac{\partial}{\partial x^i}\left(u^j \overset{0}{\mathbf{g}}_j\right) = u^j|_i \overset{0}{\mathbf{g}}_j. \tag{6}$$

Then, the following transformation of basis vectors can be obtained

$$\mathbf{g}_i = F_i^j \overset{0}{\mathbf{g}}_j, \tag{7}$$

where F_i^j is a linear differential transformation function and can be described as

$$F_i^j = \delta_i^j + u^j|_i. \tag{8}$$

where δ_i^j is the Kronecker-delta. The covariant derivative $u^j|_i$ of displacement is expressed as

$$u^j|_i = \frac{\partial u^j}{\partial x^i} + \overset{0}{\Gamma}{}_{ik}^{j} u^k, \tag{9}$$

where $\overset{0}{\Gamma}{}_{i\,k}^{j}$ is known as the Christoffel symbol of the second kind [56], and can be written as [51,54]

$$\overset{0}{\Gamma}{}_{ik}^{j} = \frac{1}{2}\overset{0}{\mathbf{g}}{}^{jl}\left(\frac{\partial \overset{0}{g}_{li}}{\partial x^k} + \frac{\partial \overset{0}{g}_{lk}}{\partial x^i} - \frac{\partial \overset{0}{g}_{ik}}{\partial x^l}\right). \tag{10}$$

It should be pointed out that $\overset{0}{\mathbf{g}}_i$ and $\mathbf{g}_i$ represent two very important local basis vectors; the stretch and rotation of a deformable body are reflected precisely through the transformation of these vectors.

On the other hand, the S-R decomposition theorem [49–54] states that any invertible linear differential transformation function **F** yields a unique additive decomposition:

$$\mathbf{F} = \mathbf{S} + \mathbf{R}, \tag{11}$$

where **S** is a symmetry sub-transformation representing the strain tensor and is positive definite and is called Chen strain, and **R** is an orthogonal sub-transformation representing the local mean rotation tensor.

The strain tensor is

$$S_j{}^i = \frac{1}{2}\left(u^i|_j + u^i|_j^{\mathrm{T}}\right) - L_k{}^i L_j{}^k (1 - \cos\theta), \tag{12}$$

and the rotation tensor is

$$R_j{}^i = \delta_j{}^i + L_j{}^i \sin\theta + L_k{}^i L_j{}^k (1 - \cos\theta), \tag{13}$$

where $L_j{}^i$ is the unit vector of the rotation axis, and $u^j|_i$ is the displacement gradient. The superscript T denotes the transpose, and the notation "$|_i$" represents the covariant derivative with respect to $\overset{0}{\mathbf{g}}_i$. And $L_j{}^i$ can be written as

$$L_j{}^i = \frac{1}{2\sin\theta}\left(u^i|_j - u^i|_j^{\mathrm{T}}\right). \tag{14}$$

The mean rotation angle θ is determined by the following formula

$$\sin\theta = \frac{1}{2}\sqrt{\left(u^1|_2 - u^1|_2^{\mathrm{T}}\right)^2 + \left(u^2|_3 - u^2|_3^{\mathrm{T}}\right)^2 + \left(u^1|_3 - u^1|_3^{\mathrm{T}}\right)^2}. \tag{15}$$

For two-dimensional problems, Eq. (15) reduces into

$$\sin\theta = \frac{1}{2}\left(u^1|_2 - u^1|_2^{\mathrm{T}}\right). \tag{16}$$

In addition, the strain rate $\dot{S}_j{}^i$ can be written as [51,54]

$$\dot{S}_j{}^i = \frac{1}{2}\left(V^i\|_j + V^i\|_j^{\mathrm{T}}\right), \tag{17}$$

where $V^i\|_j$ is the velocity gradient, and the notation "$\|_i$" represents the covariant derivative with respect to $\mathbf{g}_i$, in order to distinguish it from "$|_i$". It should be noted that, in accordance with the theory of tensor analysis, the corresponding physical components should be adopted in the calculation.

3. DDA based on S-R decomposition

3.1. Incremental governing equation

Based on the S-R decomposition, the principle of virtual power can