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Stress intensity factors for a Brazilian disc with a central crack subjected to compression

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ABSTRACT

A closed-form solution for stress intensity factors (SIFs) derived using the weight function method is developed to study the effects of crack length, loading type and friction on center-cracked Brazilian disc (CCBD) specimens. The proposed method is suitable for both opening- and closing-mode cracks. The effects of loading angle, friction coefficient and loading type (concentrated load, partially distributed pressure and confining pressure) on the SIFs are studied. The results are compared to available finite element results. It is found that the mode I SIF decreases as the relative crack length, partially distributed pressure angle and confining pressure increase. An increase in friction causes a decrease in the mode II SIF when the crack surfaces are in contact with each other. The critical crack inclination angles of both pure mode II loading (K_I=0) and crack sticking (K_{II}=0) decrease as the relative crack length and confining pressure increase. The critical crack inclination angle of crack sticking also decreases with increasing friction. When sticking conditions arise on the crack surfaces, the singular stress at the crack tip disappears. Confining pressure enhances the likelihood of sticking behavior in the crack.

1. Introduction

The test of subjecting a Brazilian disc specimen to a diametrical compressive load is currently a unique tool for easily determining the tensile strength of a rock material. To study the fracture resistances and fracture paths of rocks, center-cracked Brazilian disc (CCBD) specimens subjected to diametrical or partially distributed compression are most frequently used to study the characteristics of crack growth [1–7]. A CCBD specimen provides a simple means of measuring these values from rock cores because of its simple geometry and loading configuration. Furthermore, mixed-mode stress states ranging from pure mode I to pure mode II loading can be achieved in such a specimen by appropriately selecting the crack inclination angle and relative crack length, as shown in Fig. 1, in which the specimen in (a) is loaded with a diametrical concentrated compressive force and the specimen in (b) is loaded with a partially distributed compressive pressure. The crack extension behavior is governed by the stresses in the vicinity of the crack tip, i.e., the singular stress as characterized by stress intensity factors (SIFs). A widely used solution for determining the SIFs in a cracked Brazilian disc loaded under diametrical compression was presented by Atkinson et al. [2] in the form of an infinite series. Dong et al. [8] introduced a solution for a CCBD specimen under uniform radial pressure, also in the form of an infinite series. Awaji and Sato [9] used CCBD specimens to study combined mode I and mode II fracturing, in which the SIFs in the disc test were numerically calculated using the boundary collocation procedure and the dislocation method. Al-Shayea et al. [10] experimentally studied the fracture toughness behavior of limestone at elevated temperatures and confining pressures using CCBD specimens under diametrical compression and observed that the mode I fracture toughness (K_{IC}) was linearly proportional to the confining pressure. Khan and Al-Shayea [6] studied the effects of the testing method and specimen geometry on the fracture toughnesses measured for specimens collected from a limestone rock formation outcropping in the Central Province of Saudi Arabia. They found that the specimen diameter, crack type, and specimen type significantly affected the fracture toughness.

When a CCBD specimen is subjected to a diametrical or partially distributed pressure load, the lips of the crack may be forced into contact with each other, resulting in negative mode I SIFs for a specific crack inclination angle and relative crack length. Jia et al. [11] studied the influence of the strain rate on mixed-mode fracturing using optical methods. They reported that for specific crack inclinations, the loading mode of the crack became compressive, leading to crack closure. Al-Shayea [1] studied crack propagation trajectories under mixed-mode conditions and also concluded that compressive stresses appeared at the crack tips, resulting in crack closure, for crack inclination angles greater than 30°. Theocaris and Sakellariou [12] explained the distinction between a crack and a slit (with a finite distance between

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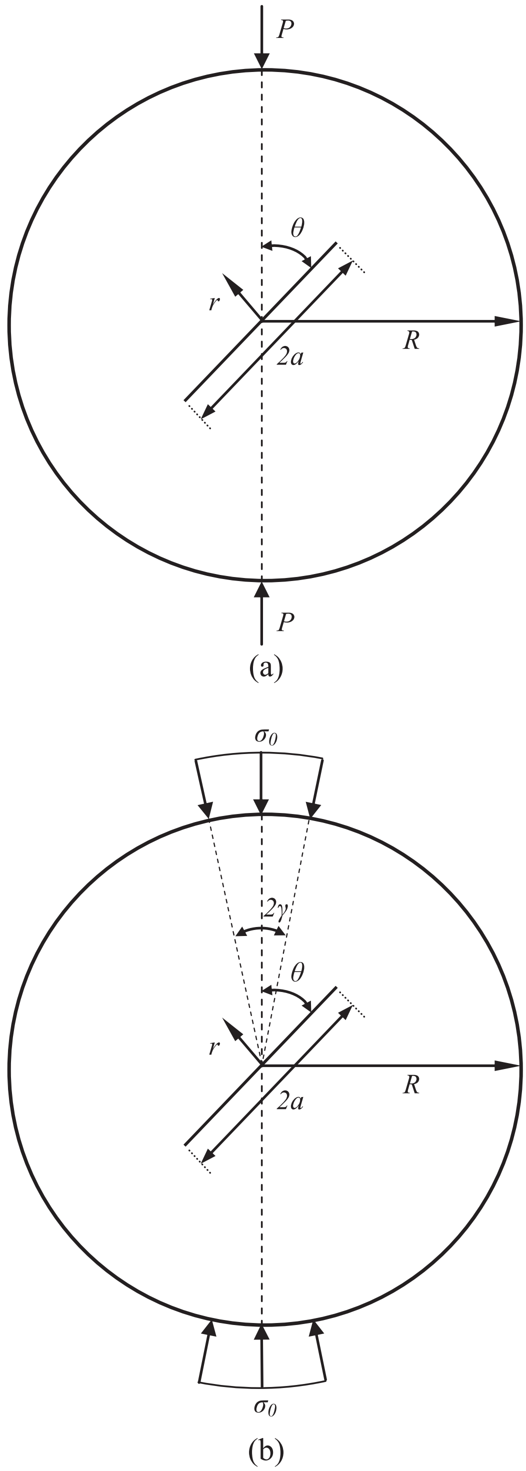

Fig. 1. Diametrical (a) and pressure (b) compression tests of a Brazilian disc specimen with an internal crack (disc thickness: B).

its lips) in a Brazilian disc by stating that for mathematical cracks (without any gap between its sides), the negative K_I-mode is impossible because inward movement of the crack lips is not allowed; furthermore, the presence of shear stresses would necessitate sufficient development of the K_I-tensile mode. Markides et al. [13] proposed closed-form expressions for the SIFs in the case of a Brazilian disc with a short central crack that are valid for both opening- and closing-mode cracks.

For a closed crack under shear and compressive loading, the friction on the crack surfaces strongly affects the mechanical behavior of the rock. In the experimental work on Griffith cracks, the friction on the crack surfaces was of little concern because a Griffith crack is an open slit. However, for a mathematical crack in linear elastic fracture mechanics (LEFM), it is assumed that there is no gap between the crack faces, as in the case of the specimens prepared by Lee and Ravichandran [14], who developed a reliable experimental method of preparing specimens with a central crack in which the crack surfaces are in contact with the desired friction coefficient. They experimentally measured the failure loads for cracks with different orientation angles and surface roughnesses, and the results were in close agreement with the theoretical predictions for compressive failure. Singh and Zimmerman [15] noted that two opposing crack faces tend to come into contact when placed under a compressive load; frictional forces will arise if the traction along the crack faces contains a non-zero component. They incorporated a realistic friction model into the formulation of the problem of the friction on compression-loaded crack faces based on the Griffith–McClintock–Walsh model. Cai [16] numerically studied the effect of the frictional resistance of a pre-existing crack on fracture initiation and propagation in a Brazilian disc. Dorogoy and Banks-Sills [17] employed a full curvilinear transformation to study the effects of contact and friction on Brazilian disc specimens that contained a crack and were subjected to concentrated loads at crack inclination angles ranging from 0° to 90°. The effects of the crack inclination angle and friction coefficient on the SIFs and the contact length were studied. The results indicate that when sticking conditions arise in the contact zone, an increase in the coefficient of friction results in an increase in the sticking region within the contact zone.

From a survey of the literature, it can be concluded that many studies of crack extension in CCBD specimens have focused on closed cracks, whereas relatively few have theoretically addressed the effects of friction on the SIFs in CCBD specimens. When a CCBD specimen is in a state of pure mode II loading and is subjected to diametrical or pressure compression, the surfaces of the ideal crack (without a gap between the crack surfaces) are in contact with each other and normal stress and frictional resistance act on the crack surfaces. Although CCBD specimens have the potential to be used as standard test specimens for mixed-mode rock fracture studies, a review of the literature indicates that the available results reported for CCBD specimens are not sufficient for addressing the extension of closed cracks. Atkinson et al. [2] presented an explicit formula to calculate the SIFs, but they did not perform a detailed study of the mechanical behavior of closed cracks in CCBD specimens. Therefore, the purpose of this investigation is to theoretically study the SIFs in CCBD specimens subjected to various loading conditions, with a particular focus on the effects of contact and friction on the mode II SIF. In contrast to the numerical analyses of Cai [16] and Dorogoy and Banks-Sills [17], this study present explicit formulas for K_I and K_{II} that are valid for any crack orientation angle in a CCBD specimen.

2. Stress distribution in a Brazilian disc specimen

Atkinson et al. [2] formulated a set of equations for the stress state of an uncracked disc under diametrical compression under the assumption of a homogeneous, isotropic, and linearly elastic material. The stress components in polar coordinates are the radial stress (σ_r), tangential stress (σ_Θ), and shear stress ($\sigma_{r\Theta}$) (Fig. 1), and they are expressed in the following forms:

$$\sigma_r = 2\sigma\left[\frac{1}{2} - \frac{(1-\rho cos\theta)(cos\theta - \rho)^2}{(1+\rho^2-2\rho cos\theta)^2} - \frac{(1+\rho cos\theta)(cos\theta + \rho)^2}{(1+\rho^2+2\rho cos\theta)^2}\right] \tag{1a}$$

$$\sigma_\theta = 2\sigma\left[\frac{1}{2} - \frac{(1-\rho cos\theta)sin^2\theta}{(1+\rho^2-2\rho cos\theta)^2} - \frac{(1+\rho cos\theta)sin^2\theta}{(1+\rho^2+2\rho cos\theta)^2}\right] \tag{1b}$$