International Journal of Rock Mechanics & Mining Sciences 93 (2017) 260–268

Contents lists available at ScienceDirect

International Journal of Rock Mechanics & Mining Sciences

journal homepage: www.elsevier.com/locate/ijrmms

Wellbore stability analysis using strain hardening and/or softening plasticity models

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ARTICLE INFO

Keywords:
Elastoplastic analysis
Wellbore stability
Strain hardening and/or softening
Drucker-Prager
Stress path
Critical mud pressure

ABSTRACT

In this paper, the general shear strain hardening and/or softening Drucker-Prager models based on the common triaxial compression test results have been introduced to model the mechanical behaviour of rock formations. These elastoplastic models are then adopted to develop rigorous analytical solutions for the drained wellbore drilling problem subjected to in-plane isotropic stress field, and to further predict the borehole collapse failure. The illustration numerical examples show the distributions of stress components and the progressive development of the plastic deviatoric strain, in addition to the effective stress trajectory for a rock point at the borehole surface due to the wellbore drilling. The advantage of the desired wellbore drilling curve presented lies in the fact that it can track the deformation responses of the borehole surface from elastic to elastoplastic states and down to the peak and ultimate/residual conditions, thus offering much more flexibility and more appropriate mud pressure design over the conventional elastic-brittle models. The critical (minimum) mud pressures, predicted by the current elastoplastic analysis based on different wellbore instability criteria, are compared with the value corresponding to the elastic theory. The solution procedure proposed can also be utilized for the tunnelling analysis and extended to the design of lining required to stabilize a tunnel.

1. Introduction

The current analytical solutions to wellbore stability analyses are mostly based on the linear elastic-brittle or poroelasticity theory, for which the borehole collapse is assumed to occur whenever the elastic/poroelastic stress state at a point surrounding the wellbore violates some failure criteria of the rock formation.[1–6] The elastic/poroelastic model is simple yet well known to be overly conservative in predicting the minimum mud weight (pressure) required to maintain the borehole stable, especially for wellbore drilling through the soft rocks.[7,8] To overcome this disadvantage usually a more advanced elastoplastic constitutive model needs to be considered, to incorporate realistically the nonlinear hardening and/or softening behaviour of the rocks, so as to predict the wellbore deformation most accurately and therefore a much less conservative estimation of the critical mud weight.

Stress and strain analyses for the wellbore drilling problem using elastoplasticity models are generally conducted through numerical methods.[7,9,10] Nevertheless, attempts have also been made in the past to obtain analytical plasticity solutions for such fundamental cavity problem, though quite limited and usually in an approximate manner. For example, Graziani and Ribacchi[11] proposed an analytical approach to determine the state of stress and strain for a circular opening excavated in a rock mass, where the rock is assumed to obey the non-associated strain softening Mohr-Coulomb model with the softening behaviour linked to a simple plastic shear strain through the so-called softening modulus parameter. Papanastasiou and Durban[12] made a further substantial extension to the cylindrical cavity expansion/contraction solutions for Drucker-Prager and Mohr-Coulomb geomaterials exhibiting arbitrary strain hardening behaviour. The formulations involved in their work, however, seem quite tedious and complicated. Charlez and Roatesi[13] derived an analytical solution for the wellbore stability problem under undrained condition using a simplistic Cam Clay model where the elliptical yield surface was replaced by two straight lines, and the solution is restricted to the volumetric strain hardening rocks with overconsolidation ratio less than 2. Recently, Chen et al.[14] proposed an analytical approach to predict the development and progress of the plastic zone around a wellbore drilled in linearly hardening or softening Drucker-Prager rocks under drained condition. The main drawback in Chen et al.[14] is that, to achieve the possible closed form solutions, the deviatoric and

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http://dx.doi.org/10.1016/j.ijrmms.2017.02.007
Received 26 February 2016; Received in revised form 16 January 2017; Accepted 2 February 2017
1365-1609/

mean effective stresses were treated in largely an approximate manner by enforcing both the two stress invariants independent of the axial stress.

In this paper, the general Drucker-Prager elastoplastic models incorporating the strain hardening and/or softening behaviour of rocks are used to investigate the wellbore stability problem in a rigorous way. The strain hardening behaviour is defined as a gradual increase in shear resistance (yield stress) with the development of plastic strain, while the strain softening refers to a progressive loss of shear resistance after a peak strength has been reached.[7,14,15] Such stress-strain features are commonly observed in the responses of rocks subject to triaxial compression testing.[8,16,17] By adopting the small strain deformation in the elastic region while large strain deformation in the plastic region, the wellbore drilling problem under drained condition is formulated as a system of first order ordinary differential equations in Lagrangian form, for any material point located in the plastic zone. The basic unknowns, i.e., radial, tangential, and vertical stresses, volumetric strain, and/or plastic shear strain, therefore can be exactly solved without much difficulty. As an illustration example, the distributions of the three stress components as well as the plastic shear strain evolution are presented against the radial distance, and the mean-deviatoric stress path for a rock point at the wellbore surface is also examined. The critical mud pressures required to prevent borehole collapse, predicted by the current elastoplastic analysis based on different wellbore instability criteria, are found to be in general well below the value predicted from the elastic theory.

2. Strain hardening and/or softening Drucker-Prager models

The strain hardening and/or softening Drucker-Prager models are natural extensions of the ideal Drucker-Parger model which provide better match to actual shearing behaviour of rocks. The typical experimental results for the behaviour of rock samples under triaxial compression conditions have been reported by many researchers,[7,8,16,17] as illustrated in Fig. 1. It is evident that the stress-strain relations are nearly linear up to certain (yield) stress levels, beyond which permanent deformations start to occur. The rock then either strain hardens continuously with the development of plastic strain towards an asymptotic limit value of stress (see Fig. 1a), or first strain hardens until a peak stress is reached and thereafter strain softens approaching its residual value (Fig. 1b). From a microstructure point of view, such experimentally observed strain hardening and/or softening material responses may be attributed to the rearrangement of the particles, and to the generation and growth of microcracks in the deformed rocks. The former allows frictional sliding at the grain-size level and thus reflecting a frictional hardening effect, while the latter may reduce the shearing resistance as a result of the formation of localized shear bands.[7] This is conceptually analogous to the frictional plastic hardening and/or softening, and can be introduced in the Drucker-Prager model by the yield function as follows

$$F(p', q, \beta) = q - \tan\beta \cdot p' \tag{1}$$

where p' and q are the mean effective stress and deviatoric stress, defined as

$$p' = \frac{1}{3}(\sigma_r' + \sigma_\theta' + \sigma_z') \tag{2}$$

$$q = \sqrt{\frac{1}{2}[(\sigma_r' - \sigma_\theta')^2 + (\sigma_r' - \sigma_z')^2 + (\sigma_\theta' - \sigma_z')^2]} \tag{3}$$

with σ_r', σ_θ', and σ_z' denoting effective radial, tangential, and vertical stresses, respectively, which are the three principal stresses for axisymmetric cylindrical wellbore problem; β is the Drucker-Prager fiction angle which as a hardening/softening parameter essentially controls the size of the current yield locus in the $p'-q$ plane, see Fig. 2. Note that in writing Eq. (1), the contribution from the cohesion

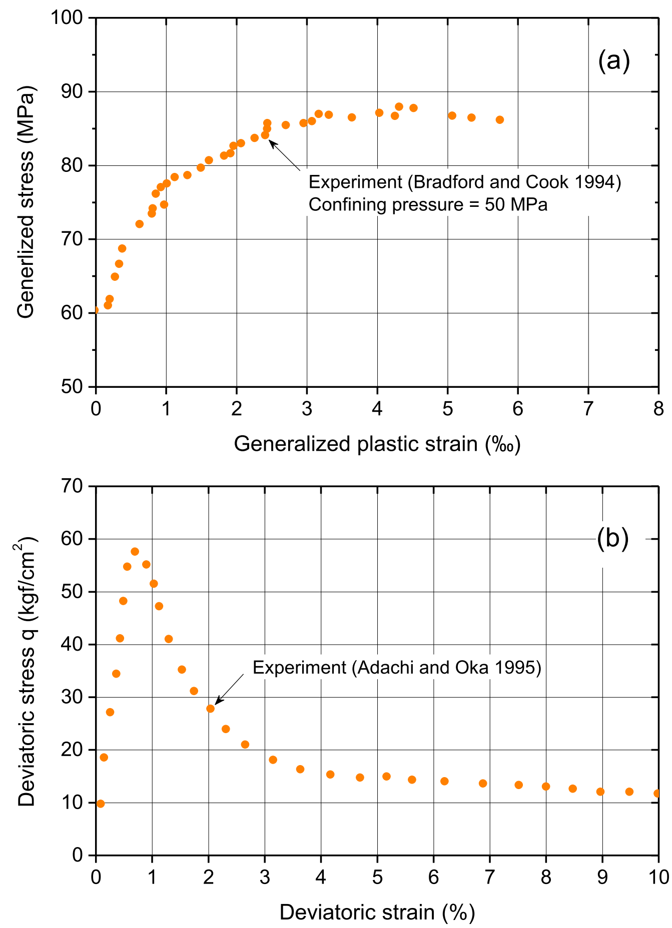

Fig. 1. Typical stress-strain relationship under triaxial compression tests: (a) strain hardening (after Bradford and Cook 1994); (b) strain hardening-softening (after Adachi and Oka 1995).

strength has been neglected for simplicity.

For a rock exhibiting a monotonic strain hardening behaviour as shown in Fig. 1a, the Drucker-Prager model requires fives material parameters: G, the elastic shear modulus; ν, the drained Poisson's ratio; β_i and β_f, the friction angles corresponding to the initial yield and failure loci, respectively (Fig. 2a); and c, a rock parameter which relates the slope of the yield locus, $\tan\beta$, hyperbolically to the accumulated deviatoric plastic strain, ε_q^p, as follows[18]

$$\tan\beta = \frac{\varepsilon_q^p}{c + \varepsilon_q^p}(\tan\beta_f - \tan\beta_i) + \tan\beta_i \tag{4}$$

where

$$\varepsilon_q^p = \int d\varepsilon_q^p = \int \frac{\sqrt{2}}{3}\sqrt{(d\varepsilon_r^p - d\varepsilon_\theta^p)^2 + (d\varepsilon_\theta^p - d\varepsilon_z^p)^2 + (d\varepsilon_r^p - d\varepsilon_z^p)^2} \tag{5}$$

with $d\varepsilon_q^p$ denoting the plastic deviatoric strain increment, and $d\varepsilon_r^p$, $d\varepsilon_\theta^p$, and $d\varepsilon_z^p$ the plastic strain increments in r, θ, and z directions, respectively.

Similarly, for a rock which first strain hardens and then strain softens corresponding to Fig. 1b (for convenience, it will be referred to as strain hardening-softening throughout the rest of this paper), the following relationship is proposed

$$\tan\beta = \tan\beta_i + A\frac{B(\varepsilon_q^p)^2 + \varepsilon_q^p}{1 + (\varepsilon_q^p)^2} \tag{6}$$

where A and B are the two plastic material parameters used to accommodate the experimental data.[15]