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Effects of small-amplitude periodic topography on combined stresses due to gravity and tectonics

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ABSTRACT

Topographic perturbations of gravitational body forces and horizontal tectonic stresses can be substantial, non-intuitive, and important in terms of subsurface engineering and rock fracture near the surface of the Earth. For (co)sinusoidal topography where the amplitude (A) is small relative to the wavelength (L), adjustments to published plane strain (two-dimensional) approximate elastic solutions for stresses in uniform, isotropic rock allow effects of gravity and a uniform regional horizontal stress (T) to be distinguished. These first-order solutions contain a characteristic stress and three geometric terms, one that varies linearly with elevation, one that decays exponentially with depth, and a (co)sinusoidal term; elastic moduli do not enter the solutions. The first-order solutions are useful approximations for $A/L < 0.04$. Both gravity and regional compression yield a compression parallel to the surface at ridge crests. Gravity, by itself, causes a localized horizontal tension below valley bottoms. Regional horizontal compression, by itself, contributes a localized vertical tension beneath ridge crests. If T is about an order of magnitude less compressive than ρgA, where ρ is rock density, and g is gravitational acceleration, then effects of gravity dominate effects of the regional compression near the topographic surface. These conditions promote opening of vertical fractures at valley bottoms. Conversely, if T is about an order of magnitude more compressive than ρgA, then effects of regional compression dominate the effects of gravity near the topographic surface. These conditions promote the opening of sheeting joints, macroscopic fractures that open near to and essentially parallel to the topographic surface.

1. Introduction

Effects of topography on stresses in rock in the shallow subsurface have drawn research interest for myriad reasons for decades.[1–14] For example, topographic stress perturbations affect tunnel safety,[15] slope stability,[16] volcanic processes,[17–19] and groundwater flow.[20,21] Topographic stress perturbations are likely to be critical in the formation of hydraulic fractures in certain areas, as well as in the formation of sheeting joints, opening mode fractures at shallow depths that essentially parallel the topographic surface.[22,23] In spite of the widely recognized importance of topography on the near-surface stress field and many advances in numerical modeling, considerable room remains for understanding basic factors of how topography perturbs ambient stresses in the shallow subsurface.

Topographic perturbations of stresses arising from gravity and tectonic loads have drawn particular attention. Although the topographic surface of the Earth clearly is three-dimensional, four two-dimensional theoretical treatments currently stand out for illuminating how topography perturbs gravitational and tectonic stresses. Holzhausen[2] and Savage et al.[5] formulated solutions for topographic perturbations of the stress field arising from horizontal tectonic stresses. Savage and Swolfs[6] and Haneberg[24] developed corresponding solutions for the topographic perturbations of gravitational body forces. Holzhausen[2] and Haneberg[24] provided first-order approximate solutions for small amplitude periodic topography, whereas Savage et al.[5] and Savage and Swolfs[6] derived exact solutions for bell-shaped topography. All of these solutions treat rock as a uniform, isotropic, isothermal linear elastic material. They provide benchmarks for numerical solutions, are relatively simple, and can be used to isolate critical length scales and characteristic stresses. Somewhat surprisingly, however, the relative contributions of gravitational and tectonic stresses to the total stress field do not seem to have been considered.

This contribution focuses exclusively on two-dimensional theoretical considerations of how periodic topography of small amplitude perturbs gravitational stresses and regional horizontal stresses. The amplitude (A) is considered to be small if it is small relative to the wavelength (L). No closed form exact solutions have been discovered for these problems. The two-dimensional, plane

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strain, linear elastic solutions considered here are for a vertical cross section perpendicular to the horizontal axis of periodic ridges and valleys. The displacements perpendicular to the plane of interest are zero, the shear stresses parallel to the cross section plane are zero, and the in-plane stresses are a function of the in-plane coordinates only. Even though rock masses exhibit some heterogeneity, anisotropy, and non-elastic behavior (e.g., Leith et al.[12]) linear elastic solutions for homogeneous, isotropic, isothermal bodies nonetheless serve as a valuable starting point for understanding the stresses at shallow depths beneath rock slopes (i.e., depths less than the topographic relief). The solutions treated here are based on the small amplitude approximations for periodic topography of Holzhausen[2] and Haneberg.[24] These solutions describe a broad range of topographic forms with only two parameters, provide useful insights that are difficult to extract from numerical solutions, can be evaluated quickly on a computer and hence are useful for exploratory analyses, and can be used to develop solutions for two-dimensional topography of arbitrary shape using Fourier series methods. Small modifications to these solutions allow the topographic perturbations of gravitational stresses and tectonic stresses to be better distinguished and compared in new ways that provide useful insights.

This treatment begins by reviewing the ambient stress fields that arise in the absence of topography from just gravity and just a uniform tectonic load. The crux of the manuscript follows: a suite of first-order solutions for near-surface topographic stresses that clearly distinguishes between the topographic effects on gravitational body forces and on a uniform regional horizontal stress field for periodic cosinusoidal topography of small amplitude. These solutions are examined separately and then jointly to identify the relative contributions of gravitational stresses and topographic stresses to near-surface stresses over a broad range of conditions. To gauge the accuracy of the first-order solutions, they are compared against numerical solutions that reproduce exact solutions. The results have a variety of practical applications, especially for understanding near-surface fracturing, but the main thrust is to develop general insights into the effect of low amplitude periodic topography on gravitational and tectonic stresses.

2. Ambient stress fields

2.1. Ambient gravitational stress field

An infinite number of theoretical plane strain stress fields honor the equations of equilibrium and compatibility[25] and account for gravitational body forces. One warrants special attention. In the absence of any lateral stresses, the ambient stresses due exclusively to the weight of a body of density ρ under a gravitational acceleration g are[25]:

$$\sigma_{xx}^{g,0} = 0, \tag{1}$$

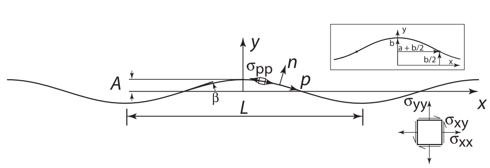

Fig. 1. Geometry of periodic topography of wavelength L and amplitude A. The x-axis is horizontal and the y-axis points up. The mean elevation of the topographic surface is at $y=0$. The n-axis is normal to the surface and the p-axis parallels the surface. The slope angle is given by β. The stress components shown are positive. The inset figure shows the bell-shaped topography of Savage et al.[5] defined by the topographic relief (b) and a width parameter (a), with inflection points at $x = \pm(a+b/2)$, $y = b/2$.

$$\sigma_{yy}^{g,0} = \rho g y, \tag{2}$$

$$\sigma_{xy}^{g,0} = \sigma_{yx}^{g,0} = 0, \tag{3}$$

where y is positive in the upward direction, $y=0$ coincides with the horizontal ground surface, and the x-direction is horizontal (Fig. 1). Normal stresses are considered positive here if tensile (Fig. 1) and negative if compressive. This is consistent with the convention of Holzhausen,[2] Savage et al.,[5] and Haneberg.[24] The stress field described by Eqs. (1)–(3) is referred to here as the ambient gravitational stress field, with superscripts g and 0.

Two other theoretical reference stress fields associate horizontal stresses with gravitational stresses. One example is that of lithostatic stress, where the horizontal normal stress equals the vertical normal stress[26]:

$$\sigma_{xx}^{lithostatic} = \rho g y, \tag{4}$$

$$\sigma_{yy}^{lithostatic} = \rho g y, \tag{5}$$

$$\sigma_{xy}^{lithostatic} = \sigma_{yx}^{lithostatic} = 0. \tag{6}$$

Anderson[26] noted that this is a convenient reference state but “will not often happen in nature.” Another theoretically permissible field involves horizontal stresses that vary linearly with depth. Where no horizontal displacement occurs[5,27] the vertical gradient in the horizontal stress can be expressed in terms of Poisson's ratio (ν), yielding the following expressions:

$$\sigma_{xx}^{lateral\ confinement} = \frac{\nu}{1-\nu}\rho g y, \tag{7}$$

$$\sigma_{yy}^{lateral\ confinement} = \rho g y, \tag{8}$$

$$\sigma_{xy}^{lateral\ confinement} = \sigma_{yx}^{lateral\ confinement} = 0. \tag{9}$$

McGarr and Gay[28] noted that this theoretical stress field for laterally confined rock, though permissible, “has rarely been observed.”

Of the theoretically possible stress fields, only the ambient stress field of Eqs. (1)–(3) occurs in the absence of horizontal normal stresses. Since a major aim here is to discriminate between how topography perturbs gravitational stresses and horizontal tectonic stresses separately, the ambient stress field of Eqs. (1)–(3) serves as the reference here for purely gravitational stresses.

2.2. Ambient tectonic stress field

The ambient tectonic stress field investigated here is for a constant lateral horizontal normal stress, the simplest non-trivial case:

$$\sigma_{xx}^{T,0} = T, \tag{10}$$

$$\sigma_{yy}^{T,0} = 0, \tag{11}$$

$$\sigma_{xy}^{T,0} = \sigma_{yx}^{T,0} = 0. \tag{12}$$

This stress field does not include a vertical gradient in the ambient horizontal tectonic stress. The focus of this manuscript, however, is for stresses at shallow depths for topography of small amplitude, and provided that the vertical gradient in the ambient horizontal tectonic stress is small relative to ρg, it can be reasonably neglected.