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Analytical solutions for an extended overcore stress measurement method based on a thermo-poro-elastic analysis

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ABSTRACT

A theoretical generalized plane-strain thermo-poro-elastic (THM) wellbore model is developed for application to the conventional overcore stress measurement method, under the situations when the thermal effect cannot be decoupled from the measured strains on the borehole walls. The THM theory developed by Coussy is used to describe the effects of both the pore pressure and the temperature on the stress and strain distributions around an inclined wellbore, which is assumed to be drilled instantly and subjected to initial stresses with arbitrary directions. The Laplace transformation is used to obtain the temperature, pressure, displacement, strain and stresses in the three-dimensional state around the wellbore. As special cases, the solutions of the above field variables are also provided for the poro-elastic (HM), thermo-elastic (TM) and pure elastic (CM) cases. Based on the THM solutions obtained, the stresses at the borehole wall, which vary with time, provide a bridge to find the in situ stresses from the measured strains at the wellbore surface. In this THM overcore stress measurement method, the temperature and the pressure (if a fluid is present) at the wellbore wall are another two parameters that need to be measured for stress inversion.

1. Introduction

Stresses around an underground opening or wellbore are of great importance in civil, mining and petroleum engineering, and have attracted a great deal of attention during the last fifty years. This paper uses examples from petroleum engineering, but the methods uses, based on coupled thermo-poro-elasticity,[1] can be easily applied to any subsurface engineering problem that requires measurement of rock stress.

In constructing wells, accurate prediction of the stresses is needed for the wellbore design to prevent potential well failures such as breakout or breakdown during drilling,[2–10] which cost the oil and gas industry billion dollars loss worldwide per year.[9] The in situ tectonic stresses can be found by using an appropriate inverse method via the strain data measured with strain gauges. For example, the overcoring technique was studied by Leeman [11] to explain the theoretical basis and to describe the design of an instrument for the determination of the complete state of stress in a borehole. The analysis of Leeman is based on the Kirsch equations.[12,13] As a large number of factors, such as temperature and pore pressure changes, will affect the stress re-distribution in rock formations at depth, a comprehensive and effective constitutive model becomes necessary.

The existing wellbore models used in stress measurement can be mainly classified into three types. The first type is based on the classical elasticity (CM) theory. The pioneering work was carried out by Kirsch [12] who studied a case where a circular hole is subjected to isotropic far-field stresses. The Kirsch solution [12] was first extended to three dimensional cases for a shaft excavated in ground by Hiramatsu and Oka [13], where the wellbore and in situ stresses are in arbitrary directions. The Kirsch solution later widely cited or mentioned in the literature [2,3,6,14,15] are more or less a special case of Hiramatsu and Oka's [13] work. Because many problems involve circular or nearly circular openings, the Kirsch solution found wide applications in rock mechanics, for example in wellbore stability and failure analysis [2–4,9,16,17] and in the determination of in-situ stresses. [11,18,19]

The second type of the wellbore models takes into account the pressure diffusion in porous media and uses the poroelasticity theory developed by Biot [20,21] and re-formulated by Rice and Cleary.[22] Afterwards, Risnes et al.,[23] Carter and Booker,[24] Santarelli et al.,[25,26] Detournay and Cheng,[27,28] Rajapakse,[29] Li [30], Ekbote et al.,[31] Coussy [32] and Chen and Yu [33] applied Biot's theory [20,21] to two-dimensional cases for a deep circular tunnels or wellbores in an isotropic poro-elastic (HM) media subjected to a non-hydrostatic stress field. In addition, Cui et al.,[34] Abousleiman and Cui,[35]

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Abousleiman and Nguyen [8] combined a loading decomposition scheme with Laplace transformation, which leads to a few fundamental problems, to obtain the analytical solutions for an infinitely long inclined borehole in isotropic or transversely isotropic HM medium.

As the thermal effect on stress and deformation has been identified to be significant,[36–38] the third type of model which takes into account thermal behaviors, i.e. thermo-poro-elasticity, has received more and more attention in for problems involving wellbore stability and hydraulic fracturing, especially for deep hot reservoirs. For example, Coussy,[1] Schiffman,[39] Palciauskas and Domenico,[40] Mctigue,[41] Kurashige [42] and Charlez [43–45] presented similar theoretical formulations for coupled thermo-poro-elasticity. Based on the above theories, Wang and Papamichos,[46,47] Zhou et al.,[48] Frydman and Fontoura,[49] Wang and Dusseault,[50] Chen and Ewy,[51] Belotserkoverts and Prevost,[52] and Wu et al. [53] studied the mechanical and thermal behaviors around a wellbore or sphere in a thermo-poro-elastic (THM) media subjected to a non-hydrostatic far-field. Li et al. [54] extended the loading decomposition method, which is used by Cui et al. [34] for inclined wellbore in HM media, to an inclined wellbore in a THM media. Abousleiman and Ekbote [55] also used a similar method to solve for stresses around a wellbore in THM transversely isotropic medium. It has to be mentioned that the solutions in the present isotropic THM model can be treated as a special case of the transversely isotropic THM model by Abousleiman and Ekbote [55].

In addition, there are other models which incorporate the chemical effects. For example, Sherwood and Bailey,[56] Ekbote and Abousleiman,[57] Nguyen and Abousleiman [58] investigated the chemical interactions effected by aqueous electrolyte, solute and ionic transport with the diffusion-deformation processes. Chemical effects are beyond the scope of the present paper.

A clear understanding of the in situ stress state at depth is of considerable value in drilling and completing wells and in applying stimulation techniques for oil and gas exploitation. Recent increasing use of horizontal wells and multistage hydraulic fracturing treatment is driving the need to develop improved stress measurement methods. Prediction of the interaction between hydraulic fractures and natural fractures during stimulation of shale gas wells requires knowledge of the 3D state of stress in the rock. The conventional overcore stress measurement methods,[11,18] which use the Kirsch equations [6,11–14] for elastic stress around a circular hole, determine the in situ stress by measuring the strain at a few points on the wellbore surface as the in situ stress is removed by overcoring. These approaches, which provide a good prediction for the in situ stresses at depth to several hundred meters below the surface or from mine or civil tunnels, need to be enhanced for application to much deep wellbores, around which there are large temperature and pore pressure differences. For example, a temperature variation $\Delta T = 100$ K can produce a stress change $\Delta\sigma = E\alpha_B\Delta T/(1-\nu_B) = 40$ MPa given the thermal expansion coefficient $\alpha_B = 8\times10^{-6}$/K, Poisson's ratio $\nu_B = 0.25$ and elastic modulus $E = 37.5$ GPa, and the thermally induced stress change is obviously not negligible.

The common steps in carrying out an overcore stress measurement are shown in Fig. 1. A small-diameter pilot hole is first drilled into the intact rock. The drilling of this pilot hole will relieve the elastic stress in the rock around the pilot hole and circulation of the drilling fluids cool the surrounding rock. After the drilling stops, the temperature and pressure in the rock around the pilot hole re-equilibrate. Ideally, the effects of pressure and temperature changes are minimized and can be neglected, as they are in the conventional overcore methods. But these time-dependent stress changes must be calculated to determine how large they are and how long they last. The next step in the overcore process is to run the stress cell instrument into the pilot hole and to establish a bond between the strain gauges and the pilot hole wall. Then the instrument is overcored to relieve the stress in the rock, inducing strains in the gauges that can be analyzed to determine the far-field in situ stress. The core will be subject to time-dependent thermo-poro-elastic stress changes during this process and sufficient time must be allowed for them to dissipate or they must be

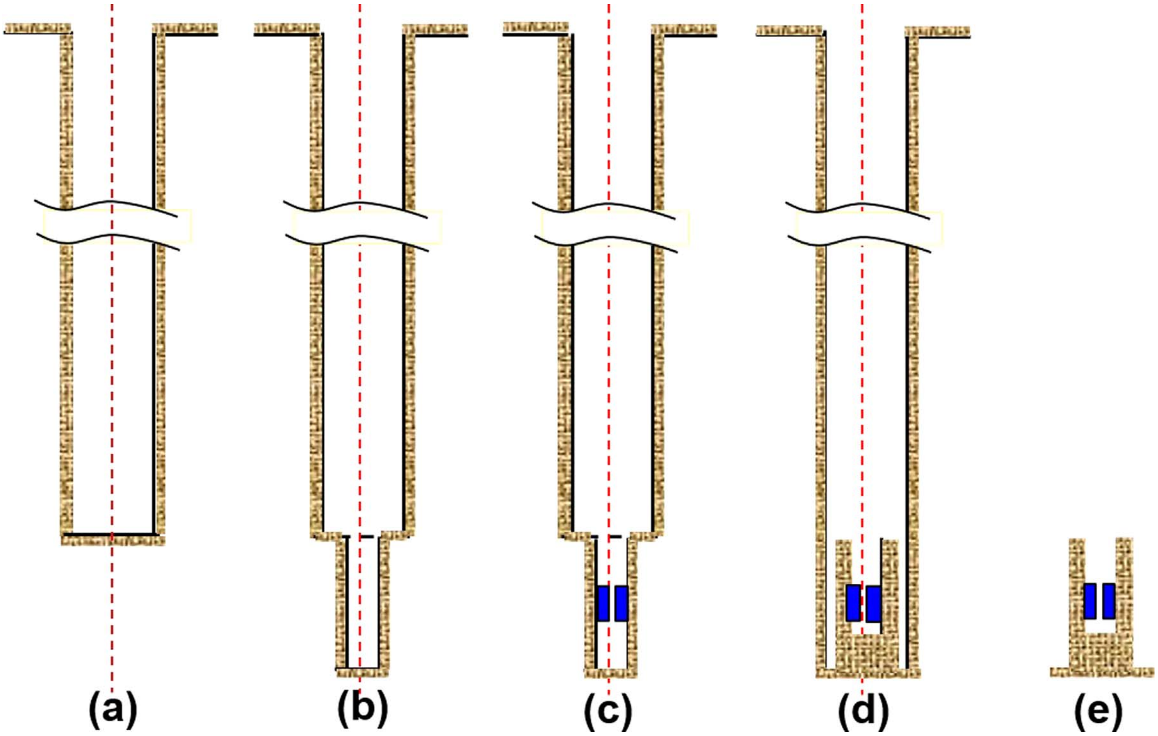

Fig. 1. Steps for overcore measurements (after [11]). (a) A main wellbore is drilled to the depth at which the stresses are to be determined; (b) a pilot hole is drilled; (c) strain gauges are set on the surface of the pilot hole and the readings are taken; (d) the pilot hole is overcored; and (e) the reading is taken after overcoring.