International Journal of Rock Mechanics & Mining Sciences 91 (2017) 1–6

Contents lists available at ScienceDirect

International Journal of Rock Mechanics Mining Sciences

journal homepage: www.elsevier.com/locate/ijrmms

A micromechanics-based damage constitutive model of porous rocks

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A R T I C L E I N F O

Keywords:
Rock
Damage constitutive model
Micromechanics
Thermodynamics
Stress-strain response

A B S T R A C T

A three-phase damage micromechanical model for rocks is developed through a combined consideration of micromechanical and thermodynamic theories. The rock is considered as a three-phase composite system, namely the rock skeleton without porosity, the initial pores, and the new cracks. The interaction among the three phases is considered. The kinetic equation of crack damage evolution is established in terms of the thermodynamic driving force derived from the reduction of Gibbs free energy of rocks. An equation that balances the thermodynamic driving force with the corresponding resistive force is obtained to calculate the damage volume fraction of crack under given stress. The stress-strain response of rocks is investigated under confining pressure or with different pore volume fraction. The theoretical results are found to be in good agreement with experimental data.

1. Introduction

In the development of national economy , underground engineering , slope engineering, oil and gas drilling engineering and so on are related to rocks. Rocks are complex natural geologic bodies, which have very complex internal structures under the external load and environment. Rocks are typical porous materials from the aspects of microstructures. There are a large number of micro cracks with random distributions in the solid phase materials. Crack propagation and damage accumulation of micro cracks will lead to the weakening of the strength and stiffness of the rocks under the external load, which make the rocks show obvious nonlinear.[1,2] The damage studies of rocks are very important for the deep understanding of the response of rock mass under the influence of thermal stress and seepage pressure. The damage constitutive relation of rocks is an important part of the study of rocks mechanics. It is also the current research difficulty problem, which should be solved in the field of geotechnical engineering.

There are a number of damage models proposed for rocks, which can be divided into two major categories: macro damage models[2–8] and micro damage models.[9–20] The macro damage models are generally based on macroscopic experimental data and formulated in the framework of irreversible thermodynamics. The damage state is characterized by scalar, vector or tensor internal variables. However, with the phenomenological approaches, the physical phenomena at different scales, which represent the origin of material damage and inelastic deformation, are not properly taken into account.

It has been found that the macroscopic mechanical behaviors of rocks strongly depend on the microstructures. Therefore, various micromechanical constitutive models have been developed to model evolving damage due to nucleation or progressive growth of random distribution microcracks.

Pensee et al.[9] have proposed a general three-dimensional micromechanical approach to modeling anisotropic damage of brittle materials such as rocks. Damage is analyzed as a direct consequence of microcracks growth. The authors of[10–12] proposed some micro–macro models which explicitly relates the macroscopic mechanical properties to the mineral composition and the porosity for Callovo-Oxfordian argillite or cement based materials. Shen and Shao[13] developed an extended micromechanical model of anisotropic rocks, and described the elastoplastic behaviors of anisotropic sedimentary rocks considering the influence of pores. Chen et al.[14] presented an empirical upper bound permeability model by considering the microstructure mechanisms, and the experimental results have been well simulated by the proposed model. Ghabezloo[15] proposed a self-consistent micromechanical model to evaluate the effective compressibility of sand stones. The sandstone microstructure is modeled by spherical inclusions with imperfect interfaces embedded in a matrix. Zhu et al.[16] developed a damage-friction coupled model based on the method of linear homogenization and the irreversible thermodynamics. An explicit function of the rock strength has been derived during the damage-friction coupled analyses. The essential feature of the damage resistance function has been described. Micromechanical based model for porous rocks like composites also can be found in.[17–19]

However, rocks are traditionally regarded as a two-phase system with the rock skeleton phase without porosity and the crack phase in the most above models. In fact, rocks usually contain some initial

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http://dx.doi.org/10.1016/j.ijrmms.2016.11.005
Received 10 October 2015; Received in revised form 9 October 2016; Accepted 2 November 2016

pores, and may also produce new cracks with increasing the load. There are interaction among the rock skeleton, initial pores and new cracks. The influence of initial pores on the rocks properties is considerable even if the pores are small.[2] Even though the rocks are considered as a multiphase system in some exiting models, a two-step homogenization method is used to analyze their linear effective properties for the sake of easy calculation.[11,20] The interaction among these phases cannot be analyzed well. It is important to further build an accurate damage model to describe microstructure-mechanical behaviors of the rocks.

Based on the equilibrium thermodynamic principles, the damage micromechanical theory has been used to predict mechanical behaviors of many materials such as shape memory alloys considered as a three-phase system. And the predicted results show good agreement with experimental results.[21,22]

In the present paper, based on the micromechanical and the thermodynamic theory, a model for rocks is developed. The rocks will be modeled as a three-phase composite system, namely rock skeleton without porosity as the matrix, the pores with a constant volume fraction as an inclusion phase, and the new cracks resulting from increasing confining pressure also as an inclusion phase. The new cracks in rocks are governed by the reduction in Gibbs' free energy of the system. Likewise, there is a resistance force associated with the nucleation and growth of new cracks. At a given level of applied mechanical stress, the driving force must be sufficient to overcome the resistance force in order to develop new cracks. The inelastic strain resulting from cracks may be determined by the criterion of general Coulomb friction.

2. Micromechanics damage constitutive modeling

Let us consider a porous rock. The representative elementary volume (REV) is considered as a three-phase system, rock skeleton matrix denoted as the 0th phase, pores denoted as 1st phase, and the new cracks denoted as 2nd phase, see Fig. 1. According to Ref.[11], the shape of pore is assumed to be spherical, and the crack as penny-shaped. The volume of the whole rock is defined as 1. The damage volume fraction of pores is constant, denoted as f_p, and the damage volume fraction of the cracks as f_c.

Under an applied far-field uniform stress $\boldsymbol{\sigma}^0$, the stress-strain relation only for matrix material is given by

$$\boldsymbol{\sigma}^0 = \mathbf{C}_s\boldsymbol{\varepsilon}^0 \tag{1}$$

where $\mathbf{C}_s$ is the stiffness tensor of the matrix.

When the three-phase system is subjected to the same far-field

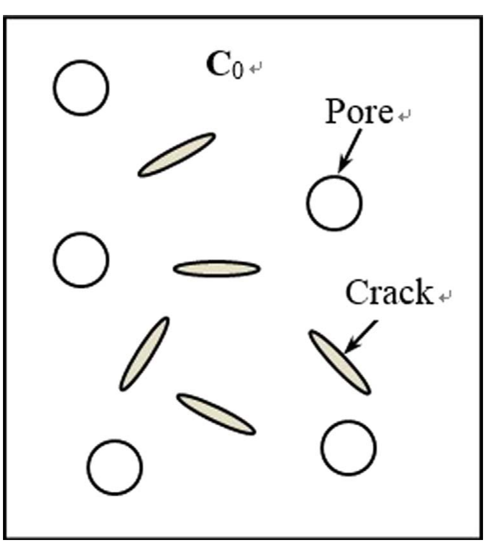

Fig. 1. Schematics of a three-phase model of rocks.

stress $\boldsymbol{\sigma}^0$, the interaction between the presences of all inclusions and the matrix may result in the average perturbed stress $\tilde{\boldsymbol{\sigma}}$ in the matrix and that corresponds to an average perturbed strain $\tilde{\boldsymbol{\varepsilon}}$. Then, its average stress and strain in the matrix are related to each other through

$$\begin{aligned}\boldsymbol{\sigma}^s &= \boldsymbol{\sigma}^0 + \tilde{\boldsymbol{\sigma}} = \mathbf{C}_s(\boldsymbol{\varepsilon}^0 + \tilde{\boldsymbol{\varepsilon}})\\ \boldsymbol{\varepsilon}^s &= \boldsymbol{\varepsilon}^0 + \tilde{\boldsymbol{\varepsilon}}\end{aligned} \tag{2}$$

The perturbed stress and strain are related by

$$\tilde{\boldsymbol{\sigma}} = \mathbf{C}_s\tilde{\boldsymbol{\varepsilon}} \tag{3}$$

The stress $\boldsymbol{\sigma}^c$ and strain $\boldsymbol{\varepsilon}^c$within the crack inclusion are also taken to be uniform. Its perturbed parts with respect to those of the matrix are denoted as $\boldsymbol{\sigma}_c'$ and $\boldsymbol{\varepsilon}_c'$, respectively.

$$\boldsymbol{\sigma}^c = \boldsymbol{\sigma}^0 + \tilde{\boldsymbol{\sigma}} + \boldsymbol{\sigma}_c' = \mathbf{C}_c(\boldsymbol{\varepsilon}^0 + \tilde{\boldsymbol{\varepsilon}} + \boldsymbol{\varepsilon}_c' - \boldsymbol{\varepsilon}_c^{tr}) = \mathbf{C}_s(\boldsymbol{\varepsilon}^0 + \tilde{\boldsymbol{\varepsilon}} + \boldsymbol{\varepsilon}_c' - \boldsymbol{\varepsilon}_c^{tr} - \boldsymbol{\varepsilon}_c^*) \tag{4}$$

where $\mathbf{C}_c$ and $\boldsymbol{\varepsilon}_c^*$ are the stiffness tensor of the crack inclusion and equivalent eigenstrain, respectively. $\boldsymbol{\varepsilon}_c^{tr}$ is the inelastic strain due to the generation of cracks. By means of Eshelby's equivalent principle, $\boldsymbol{\varepsilon}_c'$ may be written as

$$\boldsymbol{\varepsilon}_c' = \bar{\mathbf{S}}_c(\boldsymbol{\varepsilon}_c^{tr} + \boldsymbol{\varepsilon}_c^*) \tag{5}$$

where $\bar{\mathbf{S}}_c$ is the average Eshelby's tensor of cracks. Because the new cracks locate in different directions, and Eshelby tensor is different in different orientation of the cracks.

The substitution of Eq. (5) into Eq. (4) yield

$$\boldsymbol{\sigma}^c = \mathbf{C}_c[\boldsymbol{\varepsilon}^0 + \tilde{\boldsymbol{\varepsilon}} + \bar{\mathbf{S}}_c(\boldsymbol{\varepsilon}_c^{tr} + \boldsymbol{\varepsilon}_c^*) - \boldsymbol{\varepsilon}_c^{tr}] = \mathbf{C}_s[\boldsymbol{\varepsilon}^0 + \tilde{\boldsymbol{\varepsilon}} + (\bar{\mathbf{S}}_c - \mathbf{I})(\boldsymbol{\varepsilon}_c^{tr} + \boldsymbol{\varepsilon}_c^*)] \tag{6}$$

$$\boldsymbol{\varepsilon}^c = \boldsymbol{\varepsilon}^0 + \tilde{\boldsymbol{\varepsilon}} + \bar{\mathbf{S}}_c(\boldsymbol{\varepsilon}_c^{tr} + \boldsymbol{\varepsilon}_c^*) \tag{7}$$

where $\mathbf{I}$ is fourth-order identity tensor. From Eq. (6), we can get

$$\boldsymbol{\varepsilon}_c^{tr} + \boldsymbol{\varepsilon}_c^* = [(\mathbf{C}_c - \mathbf{C}_s)\bar{\mathbf{S}}_c + \mathbf{C}_s]^{-1}[-(\mathbf{C}_c - \mathbf{C}_s)(\mathbf{C}_s^{-1}\boldsymbol{\sigma}^0 + \tilde{\boldsymbol{\varepsilon}}) + \mathbf{C}_c\boldsymbol{\varepsilon}_c^{tr}] \tag{8}$$

Likewise, the average strain and stress for the pores are expressed by

$$\boldsymbol{\sigma}^p = \mathbf{C}_s[\boldsymbol{\varepsilon}^0 + \tilde{\boldsymbol{\varepsilon}} + (\mathbf{S}_p - \mathbf{I})\boldsymbol{\varepsilon}_p^*] = 0 \tag{9}$$

$$\boldsymbol{\varepsilon}^p = \boldsymbol{\varepsilon}^0 + \tilde{\boldsymbol{\varepsilon}} + \boldsymbol{\varepsilon}_p' = \boldsymbol{\varepsilon}^0 + \tilde{\boldsymbol{\varepsilon}} + \mathbf{S}_p\boldsymbol{\varepsilon}_p^* \tag{10}$$

where the corresponding stress, strain, and the Eshelby's tensor are marked with superscript or subscript 'p' to represent the values associated with the pores. From Eq. (9), we can get

$$\boldsymbol{\varepsilon}_p^* = -(\mathbf{S}_p - \mathbf{I})^{-1}(\mathbf{C}_s^{-1}\boldsymbol{\sigma}^0 + \tilde{\boldsymbol{\varepsilon}}) \tag{11}$$

Taking the volume average of the stress field under uniform stress boundary conditions, it follows that

$$\boldsymbol{\sigma}^0 = (1 - f_p - f_c)\boldsymbol{\sigma}^s + f_c\boldsymbol{\sigma}^c \tag{12}$$

The substitution of Eqs. (3) and (6) into (12) yield

$$\tilde{\boldsymbol{\varepsilon}} = \mathbf{A}_1\mathbf{C}_s^{-1}\boldsymbol{\sigma}^0 + \mathbf{B}_1\boldsymbol{\varepsilon}_c^{tr} \tag{13}$$

where

$$\mathbf{A}_1 = \{(1 - f_p)\mathbf{I} - f_c\mathbf{U}_1(\mathbf{C}_c - \mathbf{C}_s)\}^{-1}\{f_c\mathbf{U}_1(\mathbf{C}_c - \mathbf{C}_s) + f_p\mathbf{I}\} \tag{14a}$$

$$\mathbf{B}_1 = -\{(1 - f_p)\mathbf{I} - f_c\mathbf{U}_1(\mathbf{C}_c - \mathbf{C}_s)\}^{-1}f_c\mathbf{U}_1\mathbf{C}_c \tag{14b}$$

where

$$\mathbf{U}_1 = (\bar{\mathbf{S}}_c - \mathbf{I})[(\mathbf{C}_c - \mathbf{C}_s)\bar{\mathbf{S}}_c + \mathbf{C}_s]^{-1} \tag{15}$$

Substituting Eq. (13) into Eq. (11), we obtain

$$\boldsymbol{\varepsilon}_p^* = \mathbf{A}_2\boldsymbol{\sigma}^0 + \mathbf{B}_2\boldsymbol{\varepsilon}_c^{tr} \tag{16}$$

where

$$\mathbf{A}_2 = -(\mathbf{S}_p - \mathbf{I})^{-1}(\mathbf{I} + \mathbf{A}_1)\mathbf{C}_s^{-1} \tag{17}$$