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Characterization of thermal damage in sandstone using the second harmonic generation of standing waves

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ABSTRACT

This paper investigates thermal damage in sandstone using linear and nonlinear ultrasonic methods. The nonlinear ultrasonic method is based on the higher harmonic generation in non-resonant standing waves; the nonlinearity parameter is defined as the ratio of the second harmonic amplitude to the square of the fundamental amplitude. This parameter is highly sensitive to the damage evolution/microstructural changes in sandstone. The longitudinal ultrasonic velocity and dynamic elastic modulus are also measured as representative linear ultrasonic parameters. Both the linear and nonlinear parameters exhibit a peculiar reverse trend in the temperature range 100–150 °C. This effect is thought to be due to the filling of pre-existing microcracks by the clay component in sandstone, which swells and migrates with the temperature elevation. The nonlinearity parameter shows an accelerated increase at temperatures higher than 200 °C. Variations of the wave velocity and elastic modulus are up to 49%, while the nonlinearity parameter increases as much as 19 times during the thermal process. The nonlinearity parameter is more sensitive to the microstructural changes in sandstone (microcracking and clay swell and migration) due to thermal loading than the linear parameters, and thus the present nonlinear ultrasonic technique can serve as a promising tool for characterizing damage progress in rock materials.

1. Introduction

The evaluation of geotechnical engineering society for its demands from a wide range of underground engineering applications.[1–5] For example, the storage of nuclear waste often selects underground sites based on extremely strict requirements upon the integrity of surrounding rocks. However, the surface temperature of the radioactive waste container can rise up to 200 °C, which applies thermal loading on the adjacent rock mass.[2,3] Therefore, an accurate assessment of candidate rock media for thermal loading has a critical impact on the security and durability of underground deposits of nuclear waste. On the other hand, the geothermal energy extraction technology utilizes the interaction between cold water and hot-dry rock in deep underground. A fractured rock mass is more efficient for the flow of water in this application.[4] The characterization of thermally induced microcracks in rock mass is thus of great importance for the development of geothermal energy industry. Oftentimes, underground rock specimens are extracted to evaluate their mechanical properties as a most straightforward way to predict the rock behaviors during underground excavation.

As a typical heterogeneous material, rocks are present in nature with different mineral components. For rock mass subjected to an thermal load, the difference in the thermal expansion coefficients among constituting mineral components results in the microcracks at the domain boundaries which further coalesce to form a fracture network. Experimental techniques based on the ultrasonic waves have been extensively used for the assessment of thermally induced damage in different rocks. For instance, Paulsson et al. [6] and Majer et al. [7] made in-situ measurements of the change of ultrasonic velocity and attenuation on rock samples to correlate with thermal damage. Jansen et al. [8] monitored the thermally induced microcracks in rock using the ultrasonic image and acoustic emission techniques. Reuschle et al. [9] measured microstructural changes in La Peyratte granite due to heating using acoustic wave velocity.

Although experimental techniques based on the measurement of wave velocity, wave attenuation, and acoustic emission have been extensively applied for characterizing rock damage,[10–13] it has been found that these techniques are not sensitive enough to detect the early development of microcracks in rock.[14–16] The techniques based on wave velocity and attenuation of linear waves assume a linear

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constitutive relationship of the material. Microcracks give rise to a small decrease in the elastic modulus and naturally its effect is not appreciable until the damage is significant.[17] On the other hand, the experimental techniques based on the nonlinear wave phenomena (i.e., higher harmonic generation, wave modulation effect, and resonant frequency shift) have become popular for the damage characterization of various materials as the parameters from the associated nonlinear wave phenomena show high sensitivity to the damage occurrence and development of materials, particularly microcracks in the early stage.[18–25] Lately these nonlinear ultrasonic techniques have been extensively applied for damage diagnostics of rock materials as well. For instance, the nonlinearity parameter extracted from the higher harmonics has been theoretically and experimentally studied for rock samples, including the quantitative relation between the nonlinearity parameter and the wave propagation distance together with the amplitude of source waves.[26–28] Chen et al. [29] and Inserra et al. [30] used the second harmonic generation and self-demodulation phenomena to investigate damage in granite specimens. Corresponding to a wide variety of damage types, these studies commonly found that the increase of nonlinearity parameter is at least one order of magnitude higher than the variations of the traditional linear damage indicator such as wave speed and wave attenuation.

The paper aims at characterizing thermally induced damage of sandstone using a new nonlinear ultrasonic method, i.e., the second harmonic technique in standing waves. The damage index from the nonlinear ultrasonic measurements is defined to describe the progress of thermal damage of sandstone samples. In the meantime, representative linear damage parameters, namely ultrasonic pulse velocity and dynamic elastic modulus, are measured for the same rock samples. The variation of both linear and nonlinear parameters with respect to the progress of thermal damage is analyzed and the sensitivity of three experimental techniques is compared.

2. Nonlinear standing wave method

2.1. Second harmonic generation in propagating waves

The most appropriate stress-strain relationship for consolidated materials should include terms that describe both the classical and hysteretic nonlinear behaviors of these materials.[14–16,31] One-dimensional (in x-direction) constitutive relationship for a sinusoidal longitudinal motion is given in the following form [32]:

$$\sigma = E_0(1 + \beta\varepsilon + \cdots)\varepsilon + E_0\frac{\alpha}{2}[\mathrm{sgn}(\dot{\varepsilon})((\Delta\varepsilon)^2 - \varepsilon^2) - 2(\Delta\varepsilon)\varepsilon] \tag{1}$$

where E_0 is the modulus of (linear) elasticity, β and α are the quadratic and hysteretic nonlinearity parameters, is the amplitude of sinusoidal strain, sgn(x) is the sign function. The higher order polynomial terms in first part on the right hand side are from the classical atomic nonlinearity and the second part represents the hysteretic nonlinearity. Substituting Eq. (1) into the equation of motion, i.e., $\rho\partial^2u/\partial t^2 = \partial\sigma/\partial x$, one gets the following nonlinear wave equation:

$$\rho\frac{\partial^2 u}{\partial t^2} = E_0\left(\frac{\partial^2 u}{\partial x^2} + 2\beta\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2}\right) + E_0\frac{\alpha}{2}\frac{\partial}{\partial x}\left\{\mathrm{sgn}\left(\frac{\partial^2 u}{\partial t\partial x}\right)\left[\left(\Delta\frac{\partial u}{\partial x}\right)^2 - \left(\frac{\partial u}{\partial x}\right)^2\right] - 2\left(\Delta\frac{\partial u}{\partial x}\right)\frac{\partial u}{\partial x}\right\} \tag{2}$$

Under the assumption that the nonlinear behavior is much smaller than the linear behavior, the well-known perturbation solution is sought; the solution is assumed to be $u = u_1 + u_2$, where u_1 is the solution of the linear portion of Eq. (2), that is, the fundamental wave and u_2 is the second-order solution with the perturbation condition, $|u_2| < < |u_1|$. Substituting the perturbation solution into Eq. (2) and applying the perturbation condition, one obtains the following inhomogeneous wave equation for the second-order solution,

$$\rho\frac{\partial^2 u_2}{\partial t^2} = E_0\frac{\partial^2 u_2}{\partial x^2} + \beta E_0\frac{\partial u_1}{\partial x}\frac{\partial^2 u_1}{\partial x^2} + E_0\frac{\alpha}{2}\frac{\partial}{\partial x}\left\{\mathrm{sgn}\left(\frac{\partial^2 u_1}{\partial t\partial x}\right)\left[\left(\Delta\frac{\partial u_1}{\partial x}\right)^2 - \left(\frac{\partial u_1}{\partial x}\right)^2\right] - 2\left(\Delta\frac{\partial u_1}{\partial x}\right)\frac{\partial u_1}{\partial x}\right\} \tag{3}$$

It is assumed that the second order wave is so small that the magnitude of the total field can be approximated by that of the fundamental wave. Furthermore the sign change of the total wave due to the second order wave is ignored. Eq. (3) is in the form that the second-order wave is excited by the two types of nonlinearity (classical and hysteretic). The solution of Eq. (3) due to the classical quadratic nonlinearity is well known; if the fundamental wave is harmonic, e.g., $u_1 = A_1\sin(kx - \omega t)$, the second-order wave is written to be

$$u_2 = -\frac{\beta k^2 x A_1^2}{8}\cos 2(kx - \omega t) = A_2\cos 2(kx - \omega t), \tag{4}$$

where k is the wave number of the longitudinal wave at the fundamental frequency ω, and A_2 is the amplitude of the second harmonic wave. Then the quadratic nonlinearity parameter is obtained from the amplitudes of first and second harmonic waves as

$$\beta = \frac{8|A_2|}{k^2 x A_1^2}. \tag{5}$$

Let us now consider the second-order solution excited by the hysteretic nonlinearity. Introducing a new variable $X(=x - ct)$ and then using the chain rules, $\partial/\partial x = \partial/\partial X$ and $\partial/\partial t = -c\partial/\partial X$, the hysteretic excitation term is written to be

$$-\left[\frac{\partial}{\partial X}\mathrm{sgn}\left(\frac{\partial^2 u_1}{\partial X^2}\right)\right]\left(\left(\Delta\frac{\partial u_1}{\partial X}\right)^2 - \left(\frac{\partial u_1}{\partial X}\right)^2\right) - 2\,\mathrm{sgn}\left(\frac{\partial^2 u_1}{\partial X^2}\right)\frac{\partial u_1}{\partial X}\frac{\partial^2 u_1}{\partial X^2} - 2\left(\Delta\frac{\partial u_1}{\partial X}\right)\frac{\partial^2 u_1}{\partial X^2} \tag{6}$$

Now substituting $u_1 = A_1\sin kX$ and then since $\mathrm{sgn}\left(\frac{\partial^2 u_1}{\partial X^2}\right) = -\mathrm{sgn}(\sin kX)$ and $\Delta\frac{\partial u_1}{\partial X} = A_1 k$, Eq. (6) is reduced to

$$\left[\frac{\partial}{\partial X}\mathrm{sgn}(\sin kX)\right](A_1k)^2\cos^2 kX - 2\,\mathrm{sgn}(\sin kX)A_1k^3\cos kX\sin kX + 2A_1k^3\sin kX \tag{7}$$

Using the Fourier series expansion, $\mathrm{sgn}(\sin kX) = \sum_{n=1,3,5,\ldots}^{\infty}(4/n\pi)\sin nkX$, and collecting terms in order, the hysteretic excitation term is obtained in the following form:

$$E_0\frac{\alpha}{\pi}A_1^2k^3\left\{\sum_{n=1,3,5,\ldots}^{\infty}\left[\cos nkX + \left(\frac{1}{n} - \frac{1}{2}\right)\cos(n-2)kX + \left(\frac{1}{n} + \frac{1}{2}\right)\cos(n+2)kX\right] + \pi\sin kX\right\} \tag{8}$$

It is seen in Eq. (8) that the frequencies of all excitations are odd multiples of the fundamental, meaning that the hysteretic nonlinearity generates only odd harmonics. The second harmonic component in the propagating longitudinal waves is due only to the classical atomic nonlinearity.

When the material damping is not negligible, its effect on the nonlinearity determination needs to be taken into account. A common approach is to introduce the phenomenological attenuation coefficient instead of attempting to modify the governing equation. Cantrell [33] has presented the acoustic nonlinearity parameter for a material with attenuation. The fundamental and second harmonic waves are then expressed as:

$$u_1 = A_{10}\sin(kx - \omega t)e^{-\alpha_1 x} = A_1\sin(kx - \omega t), \tag{9}$$