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Elastic wave attenuation and dispersion induced by mesoscopic flow in double-porosity rocks

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ABSTRACT

The double-porosity, dual-permeability theory is employed to predict the wave attenuation and phase velocity dispersion induced by wave-induced *meso*copic fluid flow. Instead of using an up-scaled, single-porosity approximation scheme proposed by previous researchers, we develop an analytical method to exactly solve the wave equations for double-porosity materials. We first propose a new form of wave equations formulated in terms of displacements. This new form of wave equations enables us to decouple the field equations into two second-order symmetric dynamic systems, namely, the *P*-system for compressional waves and the *S*-system for shear wave. We then implement Newton iteration for solving the cubic dispersion equation for compressional waves. Finally, to understand the loss mechanism caused by mesoscopic flow, we compare the attenuation curves of the first (*P*1-wave), the second (*P*2-wave), and the third compressional waves (*P*3-wave), as well as the shear wave (*S*-wave), with the mesoscopic flow present to those with the mesoscopic flow absent. Furthermore, the effects of matrix porosity, pore-fluid viscosity, and values of fluid transport coefficient on wave attenuation are also investigated in numerical examples.

1. Introduction

Most earth materials such as rocks and sediments are generally heterogeneous and often fractured or cracked. In such materials, the pore and crack/fracture space may be filled with water, oil, or gas. When elastic waves propagate through such porous materials, fluid-related wave attenuation and dispersion occur. It is commonly accepted that the relative fluid-solid movement before the establishment of pore pressure equilibrium, known as the wave-induced fluid flow (WIFF), is the main cause of wave energy losses.

Biot's theory of poroelasticity[1,2] may be the most commonly used theoretical model to estimate the frequency-dependent attenuation and dispersion caused by WIFF. Nevertheless, during the past decades, a number of studies[3–5] have shown that the level of attenuation predicted by using the Biot's theory is drastically underestimated in the seismic band and can only be significant at frequency above 10 kHz, well outside the seismic exploration band. The principal reason for this is that in Biot's theory the porous material is assumed to be spatially homogeneous (macroscopic homogeneous) and fluid-saturated, and, as a result, only the macroscopic flow, resulting from pore pressure gradients established between the peaks and troughs of the wave, occurs, which does not produce enough loss. This mechanism is broadly known as the Biot loss and, roughly speaking, the intrinsic loss, quantified using the inverse quality factor Q^{-1}, attains Biot-loss maximum when the wavelength-scale pressure gradients just equilibrate in a wave cycle. A possible alternative to Biot's loss is the so-called "mesoscopic-loss mechanism".

Many geological materials such as reservoir rocks are generally heterogeneous and often fractured or cracked. When a compressional wave stress a material containing such mesoscopic heterogeneities (i.e., the heterogeneities exist on length scales smaller than the wavelength but greater than the pore scale), the pore pressure developed on the mesoscale drives the fluid flow between the more compliant parts of the material, e.g., cracks/fractures, and the stiffer portions, e.g., the background pores. Such wave-induced mesoscopic flow is increasingly believed to be a dominant mechanism of fluid-related attenuation in the seismic band. This mechanism is the so-called "mesoscopic-loss mechanism". Several theoretical models for wave attenuation and dispersion in porous rocks containing mesoscopic heterogeneities have been proposed during the past decades[3,6–17] (a comprehensive review of different models is provided by Müller et al.[18]). Among them, the dual-porosity, dual-permeability model may be the simplest and ideal approach to modeling fractures in porous media based on Biot's theory of poroelasticity.[9,12,13] In such models, the mesoscopic cracks/fractures are treated as a fracture network embedded in porous matrix blocks, and it is generally assumed that the fracture permeability is

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much greater than the matrix permeability, while the volume fraction of the fracture phase is much smaller than that of the matrix phase.

Early double-porosity models were initially proposed for large-scale transport problems in fissured rocks in hydrology.[19] More sophisticated models[9,12,13,17,20–29] were developed later to account for the coupling between rock deformation and flow. In particular, the double-porosity, dual-permeability models developed by Berryman and Wang[9] and Pride and Berryman[12,13] have aroused much attenuation in recent years.

Rigorously following Biot's approach, Berryman and Wang[9] make a generalization of Biot's single-porosity poroelasticity to double-porosity, dual-permeability theory to incorporate fractures. In their model, the fractures/cracks are assumed to be throughgoing joints, namely the fracture volume is all void space. The model of Berryman and Wang predicts three distinct types of compressional waves exist in a double-porosity medium. Nevertheless, the fluid flow between the matrix and fracture phases, that is the so-called "mesoscopic flow", are neglected which effect is believed to be a major cause of fluid-related attenuation in the seismic band. As a result, only Biot loss is present in their model.

Thus, more recent work of Pride and Berryman[12,13] using the volume averaging technique to derive the governing equations for double-porosity media has incorporated mesoscopic flow mechanism by allowing fluid transport between two poroelastic phases. In their model, the mesoscopic heterogeneities, which are not confined to fractures/cracks, are envisioned to be a porous continuum (e.g., a less-well consolidated sandstone), embedded within a stiffer porous host (e.g., a consolidated sandstone). But instead of solving the governing equations for the double-porosity model directly, they propose an ingenious scheme to reduce their model to an effective single-porosity Biot theory having complex frequency-dependent coefficients. Using the opproposed scheme, they demonstrate that the predicted attenuation level of the fast compressible wave ($P1$-wave) is consistent with that measured by Sams et al.[30] Furthermore, they extend the double-porosity framework to incorporate two other theoretical models, namely the squirt-flow model and patchy-saturation model.[5] Although we can apply the proposed approximation scheme to the study of $P1$-wave attenuation and dispersion, it is impossible to employ their method to investigate wave phenomena associated with the second slow wave ($P3$-wave). Thus a need remains for exactly solving the wave equations for a heterogeneous double-porosity model, which incorporates the mesoscopic-loss mechanism.

The purpose of the present paper is first to provide an analytical method to exactly solve the field equations for double-porosity materials, and then investigate the effect of mesoscopic flow on the attenuation and phase velocity dispersion of compressional and shear waves. To this end, using the solid displacement $\mathbf{u}$ and the relative fluid-solid displacements $\mathbf{w}_1$, $\mathbf{w}_2$ as independent variables, we first reformulate the governing equations of Pride and Berryman in terms of displacements. This new form of wave equations enables us to decouple the field equations into two second-order symmetric dynamic systems, each of them consisting of three coupled wave equations. Next, we substitute plane wave solutions into the obtained symmetric dynamic systems to find the dispersion relations for compressional and shear waves, and subsequently, we implement Newton iteration for solving the cubic characteristic equation for compressional waves. Finally, to understand the mesoscopic loss mechanism in double-porosity, dual-permeability models, we give examples of $P1$-, $P2$-, $P3$-, and S-waves attenuation and phase velocity dispersion.

2. Field equations

2.1. Constitutive equations

The linear constitutive relations among strain, fluid content, and pressure for isotropic materials with double-porosity can be written in the form[9,12,13,31–33]

$$\begin{pmatrix} e \\ \zeta_1 \\ \zeta_2 \end{pmatrix} = \begin{pmatrix} a_{11} & -a_{12} & -a_{13} \\ -a_{12} & a_{22} & a_{23} \\ -a_{13} & a_{23} & a_{33} \end{pmatrix} \begin{pmatrix} \sigma \\ p_{f1} \\ p_{f2} \end{pmatrix}, \tag{1}$$

where $e \equiv \partial V/V$ is the volumetric strain, $\sigma \equiv (\sigma_{xx} + \sigma_{yy} + \sigma_{zz})/3$ is the mean normal stress, p_{f1} and p_{f2} are the pore pressures in the phases 1 and 2, and the other two variables ζ_1 and ζ_2 are respectively the increments of fluid content in the two phases. The six independent coefficients a_{ij} appearing in the matrix on the right hand side are material constants.

If phase 2 is assumed to be a fracture phase with through-going joints, the constants a_{ij} are given by[31–33]

$$a_{11} = \frac{1}{K}, \tag{2}$$

$$a_{12} = 1/K_s - v_1/K_1, \tag{3}$$

$$a_{13} = v_1/K_1 - 1/K, \tag{4}$$

$$a_{22} = v_1\alpha_1/B_1K_1, \tag{5}$$

$$a_{23} = v_1/(K_s)_1 - 1/K_s, \tag{6}$$

$$a_{33} = v_2/K_f + 1/K - v_1/K_1, \tag{7}$$

where K and K_1 are the drained bulk moduli of the whole and the matrix (phase 1), K_s and $(K_s)_1$ are the unjacketed bulk moduli of the whole and the matrix, $\alpha_1 = 1 - K_1/(K_s)_1$ is the corresponding Biot-Willis coefficient, K_f is the pore fluid bulk modulus, B_1 is the Skempton's pore-pressure buildup coefficient for the matrix, and $v_2 = 1 - v_1$ is the total volume fraction of the fractures in the whole.

If, on the other hand, phase 2 is assumed to be a porous continuum, a_{ij} are determined by[5,12,32]

$$a_{11} = \frac{1}{K}, \tag{8}$$

$$a_{12} = -\frac{v_1Q_1}{K_1}\alpha_1, \tag{9}$$

$$a_{13} = -\frac{v_2Q_2}{K_2}\alpha_2, \tag{10}$$

$$a_{22} = \frac{v_1\alpha_1}{K_1}\left(\frac{1}{B_1} - \frac{\alpha_1(1-Q_1)}{1-K_1/K_2}\right), \tag{11}$$

$$a_{23} = \frac{\alpha_1\alpha_2K_1K_2}{(K_2-K_1)^2}\left(\frac{v_1}{K_1} + \frac{v_2}{K_2} - \frac{1}{K}\right), \tag{12}$$

$$a_{33} = \frac{v_2\alpha_2}{K_2}\left(\frac{1}{B_2} - \frac{\alpha_2(1-Q_2)}{1-K_2/K_1}\right), \tag{13}$$

where the Q_i are auxiliary constants given by

$$v_1Q_1 = \frac{1-K_2/K}{1-K_2/K_1}, \qquad v_2Q_2 = \frac{1-K_1/K}{1-K_1/K_2}. \tag{14}$$

In Eq. (1), the variables e, ζ_1, ζ_2 are expressed in terms of the three independent variables σ, p_{f1}, p_{f2}, and thus, this form of constitutive equations is termed the pure compliance formulation.[33] On the other hand, the variables e, ζ_1, ζ_2 can be also chosen to be independent variables and, by solving e, ζ_1, ζ_2 in terms of σ, p_{f1}, p_{f2}, the inverse relation of (1) (termed the pure stiffness formulation) can be found to be[33]

$$\begin{pmatrix} \sigma \\ p_{f1} \\ p_{f2} \end{pmatrix} = \begin{pmatrix} H + 2/3\mu & -C_1 & -C_2 \\ -C_1 & M_1 & N \\ -C_2 & N & M_2 \end{pmatrix} \begin{pmatrix} e \\ \zeta_1 \\ \zeta_2 \end{pmatrix}, \tag{15}$$

where the constant μ is the shear modulus of the drained media and other constants H, C_1, C_2, M_1, M_2, N can be written in terms of a_{ij}: