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A continuum model for tension-compression asymmetry in skeletal muscle

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ABSTRACT

Experiments on passive skeletal muscle on different species show a strong asymmetry in the observed tension-compression mechanical behavior. This asymmetry shows that the tension modulus is two orders of magnitude higher than the compression modulus. Until now, traditional analytical constitutive models have been unable to capture that strong asymmetry in anisotropic solids using the same material parameters. In this work we present a model which is able to accurately capture five experimental tests in chicken pectoralis muscle, including the observed tension-compression asymmetry. However, aspects of the anisotropy of the tissue are not captured by the model.

1. Introduction

Computational modelling of the human body has many important and practical applications, but the constitutive representation of soft tissues presents challenges. Skeletal muscle presents anisotropic and nonlinear elastic behavior as well as significant viscoelasticity and it is largely incompressible, similar to other biological soft tissues. However, in addition a strong tension-compression asymmetry has now been observed both in porcine and in chicken tissue, where the stress in tension is typically two orders of magnitude higher than in compression in all directions of loading (Takaza et al., 2013; Loocke et al., 2006; Mohammadkhah et al., 2016). This asymmetry is not captured by current constitutive modelling approaches using a single set of material parameters (Mohammadkhah et al., 2016). A recent generalisation of Ogden hyperelasticity in terms of Seth-Hill strains permits some tension/compression asymmetry (Moerman et al., 2016), but it is unclear if it can capture the extent that has been experimentally observed (Takaza et al., 2013; Loocke et al., 2006; Mohammadkhah et al., 2016). Cartilage, shape memory alloys and other materials also exhibit a degree of tension/compression asymmetry, and robust numerical approaches for modelling this response are currently in focus (Zhang et al., 2016; Du and Guo, 2014; Du et al., 2016). In this Technical Note, application of (1) the general mechanics theory of transverse isotropy in the infinitesimal strain range and (2) the recently developed What-You-Prescribe-Is-What-You-Get (WYPiWYG) formulation (Sussman and Bathe, 2009; Latorre and Montáns, 2013, 2014; Crespo et al., 2017) for the finite strain domain to the challenge of tension/compression asymmetry in passive skeletal muscle stress stretch responses are presented.

The recent experimental data on chicken pectoralis muscle are used to assess the model fitting capabilities (Mohammadkhah et al., 2016). In particular, the paper assesses the extent to which the models can simultaneously capture the tension and compression aspects of the experimental tests. Some results obtained from the WYPiWYG formulation employed in this work are initially surprising. For this reason the behavior of chicken pectoralis muscle is first assessed in the context of the classical small strain theory, but allowing for different moduli in tension and compression. Since the WYPiWYG formulation is compatible with infinitesimal theory, equivalent results are obtained when using either the small strain theory or the large strain WYPiWYG formulation. In Section 3 we present a stored energy function which simultaneously captures the behavior of the anisotropic five experiments in the finite strain regime.

2. Transversely isotropic infinitesimal strain response

2.1. Strain energy function

Consider an incompressible transversely isotropic material with different axial behavior in tension and compression along its preferred material directions. The tension/compression asymmetry holds even for small strains, so different Young's moduli for tension and compression are obtained from uniaxial testing. The isotropic plane is defined by axes 1 and 2 (direction 1 is the muscle cross-fibre direction) and the muscle fibre direction is axis 3. Then, ε_{11}, ε_{22} and ε_{33} are the axial components in preferred directions of the infinitesimal isochoric strain tensor $\boldsymbol{\varepsilon}$ and $\varepsilon_{13}^{\#} = \sqrt{\varepsilon_{13}^2 + \varepsilon_{23}^2}$ is a composite shear deformation invariant

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including the shear components ε_{13} and ε_{23}. We can exactly characterize this material with a single isochoric strain energy function of the form

$$\mathscr{W}(\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}, \varepsilon_{13}^{\#}) = \omega_{11}(\varepsilon_{11}) + \omega_{11}(\varepsilon_{22}) + \omega_{33}(\varepsilon_{33}) + 2\omega_{13}(\varepsilon_{13}^{\#}) \tag{1}$$

with $\omega_{ii}(\varepsilon)$, $i = \{1, 3\}$, including the tension/compression asymmetry effects, i.e. being piecewise bi-quadratic—subscripts c and t refer to compression and tension, respectively

$$\omega_{ii}(\varepsilon) = \begin{cases} \mu_{ii}^{c}\varepsilon^2 & \text{if } \varepsilon < 0 \\ \mu_{ii}^{t}\varepsilon^2 & \text{if } \varepsilon \geq 0 \end{cases} \tag{2}$$

and $\omega_{13}(\varepsilon_{13}^{\#})$ being quadratic—shear behavior within preferred planes is always symmetric

$$\omega_{13}(\varepsilon_{13}^{\#}) = \mu_{13}(\varepsilon_{13}^{\#})^2 = \mu_{13}(\varepsilon_{13}^2 + \varepsilon_{23}^2) \tag{3}$$

The five deviatoric moduli (material constants) μ_{11}^{c}, μ_{11}^{t}, μ_{33}^{c}, μ_{33}^{t} and μ_{13} characterize the generally bi-linear strain-stress response. This additive, fully uncoupled decomposition in terms of the small strain tensor components in preferred material directions is not a hypothesis within the incompressible infinitesimal strain setting, but rather it is a consequence of taking such a limit in which possible higher order couplings vanish.

2.2. Tension/compression uniaxial tests along fibre and cross-fibre directions

We consider both tension and compression uniaxial tests along both cross-fibre and fibre directions, i.e. four uniaxial tests from which we should be able to determine the four material constants μ_{11}^{c}, μ_{11}^{t}, μ_{33}^{c} and μ_{33}^{t}. In these cases $\varepsilon_{11} \equiv \varepsilon_1$, $\varepsilon_{22} \equiv \varepsilon_2$ and $\varepsilon_{33} \equiv \varepsilon_3$ are isochoric principal strains. From the tensile test along the cross-fibre direction 1 we have—i.e. $\varepsilon_1 > 0$

$$\sigma_1 = \omega'_{11}(\varepsilon_1) + p = 2\mu_{11}^{t}\varepsilon_1 + p \tag{4}$$

where σ_1 is the (Cauchy) stress in axis 1 and p is the pressure Lagrange multiplier associated to the incompressibility constraint $\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 0$ to be determined from the boundary conditions. In the other axes we have

$$0 = \omega'_{11}(\varepsilon_2) + p = 2\mu_{11}^{c}\varepsilon_2 + p = -2\mu_{11}^{c}\nu_{12}^{t}\varepsilon_1 + p \tag{5}$$

$$0 = \omega'_{33}(\varepsilon_3) + p = 2\mu_{33}^{c}\varepsilon_3 + p = -2\mu_{33}^{c}\nu_{13}^{t}\varepsilon_1 + p \tag{6}$$

where, according to experimental evidence (Mohammadkhah et al., 2016), we have assumed positive Poisson ratios in both axes 2 and 3 during the tensile test in axis 1, i.e. transverse contraction given by $\varepsilon_2 = -\nu_{12}^{t}\varepsilon_1 < 0$ and $\varepsilon_3 = -\nu_{13}^{t}\varepsilon_1 < 0$, so we have used the moduli μ_{11}^{c} and μ_{33}^{c} in Eqs. (5) and (6), respectively. The Lagrange multiplier p may be eliminated and the incompressibility condition (which also results in $\nu_{12}^{t} + \nu_{13}^{t} = 1$) be employed to arrive at

$$\begin{cases} \nu_{12}^{t} = \dfrac{\mu_{33}^{c}}{\mu_{11}^{c} + \mu_{33}^{c}}, \quad \nu_{13}^{t} = \dfrac{\mu_{11}^{c}}{\mu_{11}^{c} + \mu_{33}^{c}} \\[2ex] Y_1^{t} = 2\left(\mu_{11}^{t} + \dfrac{\mu_{11}^{c}\mu_{33}^{c}}{\mu_{11}^{c} + \mu_{33}^{c}}\right) \end{cases} \tag{7}$$

where Y_1^{t} is the Young modulus during the tensile test in direction 1. Equivalently, for a compression test

$$\begin{cases} \nu_{12}^{c} = \dfrac{\mu_{33}^{t}}{\mu_{11}^{t} + \mu_{33}^{t}}, \quad \nu_{13}^{c} = \dfrac{\mu_{11}^{t}}{\mu_{11}^{t} + \mu_{33}^{t}} \\[2ex] Y_1^{c} = 2\left(\mu_{11}^{c} + \dfrac{\mu_{11}^{t}\mu_{33}^{t}}{\mu_{11}^{t} + \mu_{33}^{t}}\right) \end{cases} \tag{8}$$

where ν_{12}^{c}, ν_{13}^{c} and Y_1^{c} are the Poisson ratios and the Young modulus during the uniaxial compression test in direction 1, respectively. Performing similar algebra for a test in direction 3

$$\begin{cases} \nu_{31}^{t} = \nu_{32}^{t} = \nu_{31}^{c} = \nu_{32}^{c} = \frac{1}{2} \\[1ex] Y_3^{t} = 2\left(\mu_{33}^{t} + \frac{1}{2}\mu_{11}^{c}\right) \\[1ex] Y_3^{c} = 2\left(\mu_{33}^{c} + \frac{1}{2}\mu_{11}^{t}\right) \end{cases} \tag{9}$$

where the subscripts indicate the respective axes and the superscripts t and c mean tension and compression, respectively. Considering tension and compression in the muscle fibre and cross fibre directions shows that only four of these constants are independent. From these equations we can determine μ_{11}^{c}, μ_{11}^{t}, μ_{33}^{c} and μ_{33}^{t}, to which μ_{13} is added. If we use the four Young's moduli to determine the material constants, the Poisson's ratios are automatically obtained as result. We now solve Eqs. (7)₂, (8)₂, (9)₂ and (9)₃ taking the reference Young's moduli in the underformed configuration from the experimental data from chicken pectoralis muscle, provided in Mohammadkhah et al. (2016), which we have measured approximately as

$$Y_1^{t} = 163 \text{ kPa}, \quad Y_1^{c} = 2.95 \text{ kPa}, \quad Y_3^{t} = 100 \text{ kPa}, \quad Y_3^{c} = 2.70 \text{ kPa} \tag{10}$$

The tensile response is seen to be two orders of magnitude larger than in compression for both directions, as noted in Mohammadkhah et al. (2016). Moreover, chicken muscle tissue is most compliant in the fibre direction 3 (denoted *longitudinal* therein and labelled *L*) than in the cross-fibre direction 1 (denoted *transverse* therein and labelled *T*) for both tensile and compressive applied deformation. Using the experimental yield moduli, the previous equations give the solution[1]

$$\mu_{11}^{t} = 104 \text{ kPa}, \quad \mu_{11}^{c} = -40.4 \text{ kPa}, \quad \mu_{33}^{t} = 70.2 \text{ kPa}, \quad \mu_{33}^{c} = -50.6 \text{ kPa} \tag{11}$$

which in turn result in the following (not independent) Poisson's ratios

$$\nu_{13}^{t} = 0.44 \quad \text{and} \quad \nu_{13}^{c} = 0.60 \tag{12}$$

These transverse-to-axial strain ratios are different to those actually observed in the tissue (see Table 3 in Mohammadkhah et al., 2016, namely $\nu_{13}^{t} \equiv \nu_{TL}^{t} \approx 0.83$ and $\nu_{13}^{c} \equiv \nu_{TL}^{c} \approx 0.34$). These are given for larger deformation levels and in terms of logarithmic strain ratios, hence they are not directly comparable with the present solution. These differences in computed continuum Poisson's ratios and experimental ones could be explained in part from the fact that they have been predicted using a purely continuum theory, assuming a sufficiently large scale such that the continuum principles hold, whereas in muscle the size of the specimen may be relevant in the observed behavior at larger than usual scales (Meyer and Lieber, 2011; Meyer et al., 2011; Gras et al., 2013). Furthermore, passive muscle behavior under compression is somewhat dictated by specimen size at the tissue level (Simms et al., 2017) where, additionally, the difficulty in imposing common boundary conditions for every specimen size, along with differences in fluid exudation, could have some adverse effects. Since our model captures exactly the tests presented to the model, all the mentioned discrepancies are reflected in the Poisson ratios.

The deviatoric moduli given in Eq. (11) include two positive moduli associated with both tension branches in Eq. (2) and two negative moduli associated with both compression branches in Eq. (2). Traditional isochoric hyperelastic models, based on continuously differentiable analytical hyperelastic functions, are unable to include this type of positive-tension/negative-compression asymmetry. Importantly, the solution encountered herein in an infinitesimal scenario may explain why experimental data from this specific type of skeletal muscle has not

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[1] Two more solutions are obtained, but they are rejected because yield negative Poisson ratios as a consequence, which is in contradiction with the initial hypotheses considered herein (see, for example, Eqs. (5) and (6)) based on experimental evidence (Mohammadkhah et al., 2016).