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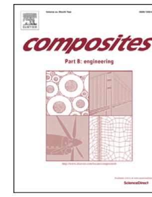

A wave-based computational method for free vibration, wave power transmission and reflection in multi-cracked nanobeams

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ABSTRACT

In this paper, the wave propagation method and the nonlocal elasticity theory are utilized to analyze the vibration, wave power transmission and reflection in multi-cracked Euler-Bernoulli nanobeams. This aim is pursued by deriving the propagation, reflection and transmission matrices and comparing the natural frequencies obtained by these matrices with the available results in the literature. Then, the nonlocal and crack-severity effects on the natural frequencies are presented for some combinations of the boundary conditions. Finally, the effects of nonlocal and crack-severity parameters on the reflected and transmitted power of a wave incident upon a crack location are studied in details for cracked nanobeams. The results obtained via the reflection and transmission matrices will provide valuable insights into the subject of wave power reflection and transmission analysis in nanoscale structures for the future. The computer coding of the proposed method is much easier than the classical vibration analysis methods for similar analyses which makes it more appropriate in implementation.

1. Introduction

Nanotechnology deals with nanoscale materials and their applications. Researchers endeavor to extol their products using two main allotropes of carbon, namely graphene sheets (GSs) and carbon nanotubes (CNTs). The CNTs have been modeled as shells, Euler-Bernoulli beam, Timoshenko beam, and elastic rod [1–5]. In the literature [1–5], the structures are assumed to be intact or free from defects. It is known that defects can change the mechanical behaviors of structures. For example, cracks, as a common defect in structural elements, can reduce the natural frequencies of the structures due to the fact that they become more flexible in presence of the cracks. Thus, the understanding of defects can improve the design of Nanoelectromechanical Systems (NEMS). There are some studies in which the effects of the defects are considered [6–10]. Longitudinal and transverse vibrations of cracked nanobeams were studied within the framework of the nonlocal Euler–Bernoulli and the nonlocal Timoshenko theories [6–10].

Different nonlocal theories have been integrated with various methods to study nanoscale structures [11–25]. In the works of Eltaher et al. [11,12], the free vibration and stability analyses of FGM nanobeams were studied via the finite element method. In another study by Phadikar and Pradhan [13], nanoplates and nanobeams were analyzed via the finite element method with a linear nonlocal formulation. Moreover, Zhang et al. [14] studied the free vibration, buckling and bending of micro/nanobeams via a hybrid nonlocal Euler–Bernoulli beam model. In an article by Civalek and Akgoz [15], free vibration of microtubules were analyzed via Differential Quadrature (DQ) method. Gürses et al. [16] analyzed the free vibration of nano annular sector plate based on the nonlocal continuum theory and the discrete singular convolution method (DSC). Demir and Civalek [17] studied the torsional and axial response of microtubules using the nonlocal continuum and nonlocal discrete model via finite element method. Akgöz and Civalek [18] introduced a new size-dependent beam model based on the hyperbolic shear deformation beam and modified strain gradient theory. Apuzzo et al. [19] proposed an enhanced model of nonlocal torsion based on the Eringen theory for nanobeam. Barretta et al. [20] proposed a gradient Eringen model for functionally graded nanorods. In another work, Barretta et al. [21] presented a fully gradient elasticity model for bending of nanobeams using a nonlocal thermodynamic approach. An enhanced version of the Eringen differential model was outlined by Barretta et al. [22]. Moreover, Barretta et al. [23] studied the transversal deflection of Timoshenko nanobeams using a nonlocal Eringen-like constitutive law described by two material length-scale parameters. Furthermore, Barretta et al. [24] proposed the first gradient nonlocal model of bending for

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Timoshenko functionally graded nanobeams based on the Eringen model.

There is also an approach called wave propagation which is separable from the above mentioned methods. This simple and non-iterative approach considers the vibrations as waves propagating through the structure and can be utilized for the vibration analysis of the structures. An important difference of the wave propagation approach from other methods is its additional ability to provide a set of reflection and transmission matrices allowing the reflected, transmitted power and energy flow of the waves in the waveguides to be gauged. This ability highlights the utility of wave propagation approach for energy reflection and transmission analysis as well as vibration analysis. This approach was mainly utilized in macrostructures. The vibration of Euler-Bernoulli beams [26], Timoshenko beams [27,28], rotating Timoshenko beam [29], curved beams [30], and variable thickness beams [31,32] are the main researches in one dimensional waveguides. The wave propagation method was also utilized for free vibration analysis of frames [33,34]. Moreover, vibrations of thin cylindrical shell [35,36], cross-ply laminated composite cylindrical shell [37], and fluid-filled cylinder [38] were investigated using the wave propagation approach. Recently, Bahrami et al. [39] used the wave propagation method to calculate the natural frequencies of circular and annular membranes. In other works, Bahrami and Teimourian have also reported the utility of this approach for composite circular [40] and rectangular membranes [41].

The literature review points out the scarcity of studies on wave reflection and vibration analysis of nanostructures through the wave propagation approach. The influence of nonlocal scale on the wave power reflection in rectangular nanoplate has been investigated by Ilkhani et al. [42] using the wave propagation method. Recently, Bahrami and Teimourian used the wave propagation method to study the small scale effect on the wave power reflection in Euler based nanobeams [43], Timoshenko nanobeams [44] and circular annular nanoplates [45]. Based on author's knowledge, there are still no studies on energy reflection and transmission in nanostructures when there exist cracks in nanostructures. The practical applications of nanostructures in the industry necessitates a simple computational wave approach for analysis of wave transmission and reflection in these structures. In the present paper, the wave propagation technique is applied to a multi-cracked nanobeam and the accuracy of the obtained natural frequency results by this method are assessed by comparing them with the results provided in the literature. Then, the nonlocal and crack-severity effects on the natural frequencies are presented for some combinations of the boundary conditions. Finally, the effects of nonlocal and crack-severity parameters on the reflected and transmitted power of a wave incident on a crack location are studied in details for the cracked nanobeams. The results obtained via the reflection and transmission matrices will provide valuable insights into the subject of energy reflection and transmission analysis in nanoscale structures for the future.

2. Mathematical formulations

2.1. Equation of motion

In nonlocal elasticity theory, constitutive equations incorporate the effects of atomic forces and small scale as material parameters [46]. The differential form of the nonlocal constitutive equation has been developed by Eringen [46] as follows:

$$\left(1-(e_0a)^2\nabla^2\right)\boldsymbol{\sigma}=\mathbf{C}:\boldsymbol{\varepsilon} \tag{1}$$

where ∇^2 denotes the Laplacian operator, $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ are the stress and strain tensors, $\mathbf{C}$ is fourth order elastic modulus tensor, a denotes the internal characteristic length, and e_0 is a constant depending on the material characteristics. The value of parameter a depends on lattice parameter, granular size and C-C bonds, and the material constant. The parameter e_0a is called the small scale. In any type of analysis, comparison of the continuum modeling results with those of atomistic ones determines the value of this parameter. Consider a cracked beam with n-1 cracks and n identical segments as shown in Fig. 1. We can write the governing equations of a nonlocal Euler–Bernoulli cracked beam model for its n segments as:

$$EI\frac{\partial^4 w_j}{\partial x^4}+\rho A\frac{\partial^2}{\partial t^2}\left(w_j-(e_0a)^2\frac{\partial^2 w_j}{\partial x^2}\right)=0,\ \ j=1,2,\dots,n \tag{2}$$

Three classical types of boundary condition can be categorized based on the equation of motion as:

$$\begin{aligned}&\text{Free}: && M=0\\ &&& V=0\\ &\text{Clamped}: && w=0\\ &&& \frac{\partial w}{\partial x}=0\\ &\text{Simply supported}: && w=0\\ &&& M=0\end{aligned} \tag{3}$$

The moment and shear force of the beam can be obtained based on the nonlocal elasticity as follows [43]

$$M=-EI\frac{\partial^2 w}{\partial x^2}+(e_0a)^2\rho A\frac{\partial^2 w}{\partial t^2} \tag{4}$$

$$V=-EI\frac{\partial^3 w}{\partial x^3}+(e_0a)^2\rho A\frac{\partial^3 w}{\partial x\partial t^2} \tag{5}$$

The general solution of Eq. (2) can be written as:

$$w_j(x,t)=\sum_{m=1}^{\infty}A_{mj}e^{i(r_jx-\omega t)},\quad j=1,2,\dots,n \tag{6}$$

in which r_j are the wave numbers, $i=\sqrt{-1}$, t is time, and ω denotes the frequency. Substituting Eq. (6) into Eq. (2) yields

$$EIr_j^{\,4}-\rho A(e_0a)^2\omega^2r_j^{\,2}-\rho A\omega^2=0,\quad j=1,2,\dots,n \tag{7}$$

The analytical solution can be obtained by solving Eq. (7) as follows:

$$\begin{aligned}r_{j(1,2)}&=\pm\gamma_j\\ \gamma_j&=\frac{1}{l}\sqrt{\frac{\beta^2k^2+\sqrt{4\beta^2+\left(\beta^2k^2\right)^2}}{2}},\quad j=1,2,\dots,n\\ r_{j(3,4)}&=\pm i\alpha_j\\ \alpha_j&=\frac{1}{l}\sqrt{\frac{\sqrt{4\beta^2+\left(\beta^2k^2\right)^2}-\beta^2k^2}{2}},\quad j=1,2,\dots,n\end{aligned} \tag{8}$$