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Nonlinear vibration of single-walled carbon nanotubes with nonlinear damping and random material properties under magnetic field

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ABSTRACT

This paper copes with the statistical dynamic behaviors of nonlinear vibration of the single-walled carbon nanotubes (SWCNTs) under longitudinal magnetic field by considering the effects of the geometric nonlinearity and nonlinear damping. Both the Young's modulus of elasticity and mass density of the SWCNTs are considered as stochastic with respect to the position to actually characterize the random material properties of the SWCNTs. Based on the theory of nonlocal elasticity, the small scale effects of the nonlinear vibration of the SWCNTs are investigated. By using the Hamilton's principle, the nonlinear governing equations of the single-walled carbon nanotubes subjected to longitudinal magnetic field are derived. The Monte Carlo Simulation, Galerkin's method and the multiple scale method are adopted to solve the nonlinear governing equation and to calculate the statistical response of the SWCNTs. Some statistical dynamic responses of the SWCNTs such as the mean values and standard deviations of the midpoint deflections are computed, the effects of the small scale coefficients, magnetic field, nonlinear damping and the elastic stiffness of matrix on the statistical dynamic responses of the SWCNTs are investigated and discussed.

1. Introduction

Carbon nanotubes (CNTs) has drawn worldwide attention because of their potential applications in the fields of physics, chemistry, electrical engineering, nano-engineering, materials science, reinforced composite structures and civil engineering [1–3]. Hence, several theoretical and experimental methods [4–6] have been utilized to calculate the mechanical properties of nanotubes. Based on the previous studies, it is found that the size effect is fairly important and plays an important role on the mechanical behavior of nanostructures when the dimensions of the structures become very tiny. Eringen [7] used the nonlocal elasticity theory to investigate the small scale effect in elasticity, he utilized the theory to investigate the lattice dispersion of elastic waves, wave propagation in composites, dislocation mechanics and fracture mechanics. Based on Eringen's nonlocal elasticity theory, the stress at a reference point in a body is considered to be a function of the strains at all the points in the near region. The above assumption is in agreement with the atomic theory of lattice dynamics and experimental observations on phonon dispersion. After the first several investigations on mechanical properties of nanotubes with the nonlocal continuum theory [8–11], lots of researches have been reported on the characteristics of buckling [12–17] and vibration [18–29] of nanotubes. Based on the previous theoretical and experimental investigations, it is detected that the mechanical behavior of nanostructures is actually nonlinear in nature when they are under large external loads [30]. Among others, many researches on nonlinear problems with nonlocal continuum theories have been reported [31–37].

By utilizing AFM test on clamped–clamped nanoropes, Salvetat et al. [38] estimated the flexural Young's modulus and shear modulus and obtained the values with 50% of error. Besides, Krishnan et al. [39] depicted the histogram distribution of the flexural Young's modulus by performing AFM test on 27 CNTs. The Young's modulus was estimated observing free-standing vibrations at room temperature using transmission electro-microscope (TEM), with a mean value of 1.3 TPa and the deviation of mean value is from −0.4 TPa to +0.6 TPa. In addition, the stochastically averaged probability amplitude for the vibration modes is calculated to obtain the root-mean-square vibration profile along the length of the tubes [40]. Therefore, uncertainty is also related to the equivalent atomistic-continuum models especially in the engineering and materials science communities. Hence, to be realistic, the

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Young's modulus of elasticity of carbon nanotubes (CNTs) should be considered as stochastic with respect to the position to actually describe the random property of the CNTs under certain conditions. Furthermore, random vibration analysis of the CNTS must be performed due to the nature of the external loadings. In the past, the random vibration or random system researches on the carbon nanotubes have been reported in some studies [41–46]. Recently, Chang [47] used the stochastic finite element method to perform the nonlinear vibration analysis on fluid-loaded double-walled carbon nanotubes under a moving load based on nonlocal elasticity theory. In his work, the Young's modulus of elasticity of the DWCNTs was considered as stochastic with respect to the position, however, the mass density of the DWCNTs was considered deterministic.

In the present study, we investigate the stochastic dynamic behaviors of nonlinear vibration of the single-walled carbon nanotubes (SWCNTs) subjected to longitudinal magnetic field by considering the effects of the geometric nonlinearity and nonlinear damping. Not only the Young's modulus of elasticity of the SWCNTs is considered as stochastic with respect to the position, but also the mass density is considered as stochastic with respect to the position to actually characterize the random material properties of the SWCNTs. Besides, the small scale effects on the nonlinear vibration of the SWCNTs are considered by using the theory of nonlocal elasticity. Based on the Hamilton's principle, the nonlinear governing equations of the single-walled carbon nanotubes subjected to longitudinal magnetic field are formulated. The Monte Carlo simulation along with the Galerkin's method and multiple scale method are used to investigate the statistical response of the SWCNTs. The effects of the small scale coefficients, nonlinear damping, magnetic field and the viscous matrix stiffness on the statistical dynamic responses of the SWCNTs are studied.

2. Governing equation of nonlinear vibration

As shown in Fig. 1, the single-walled carbon nanotubes (SWCNTs) embedded in the viscous elastic matrix with longitudinal magnetic field is modeled as a single-tube pipe which has the radius R. The thickness of the tube is h, the length is L, the Young's modulus of elasticity is $E(x)$ and the mass density of SWCNTs is $\rho(x)$. It is noticed that the Young's modulus of elasticity $E(x)$ and the mass density of SWCNTs $\rho(x)$ are assumed as stochastic with respect to the position to actually describe the random material property of the SWCNTs. The geometric shapes and sizes of the SWCNTs, the spring constant k_w, the damping coefficient c of the matrix, applied load and the longitudinal magnetic field H_x are considered as deterministic. In addition, the boundary conditions of the SWCNTs are considered as simply-supported at both ends.

Based on nonlocal elasticity theory and the formulation derived by Wang and Li [48], the nonlocal governing equations of the SWCNTs in terms of the displacements can be obtained as follows:

$$I\frac{\partial^2\left(E(x)\left(\partial^2 w/\partial x^2\right)\right)}{\partial x^2}+\left[\int_0^L\left(\frac{\partial w}{\partial x}\right)^2\frac{E(x)A}{2L}dx\right]\frac{\partial^2}{\partial x^2}\left[(e_0a)^2\frac{\partial^2 w}{\partial x^2}-w\right]=\rho(x)A\frac{\partial^2}{\partial t^2}\left[(e_0a)^2\frac{\partial^2 w}{\partial x^2}-w\right]+k_w\left[(e_0a)^2\frac{\partial^2 w}{\partial x^2}-w\right]+c\frac{\partial}{\partial t}\left[(e_0a)^2\frac{\partial^2 w}{\partial x^2}-w\right]+q_w-(e_0a)^2\frac{\partial^2 q_w}{\partial x^2} \tag{1}$$

where A is the cross area, I is the inertial moment, q_W is distributed transverse load, e_0 is a constant appropriate to each material, a is an internal characteristic length (e.g., length of C–C bond, lattice parameter, and granular distance). Based on the previous study [49], we consider the nonlinear viscous damping since it is shown more effective in suppressing the resonant peak of a nonlinear system than linear damping, meanwhile we also consider the SWCNTs is subjected to an externally applied longitudinal magnetic field, based on the formulations derived by Murmr et al. [50], the governing equation of motion of the system can be expressed as follows:

$$I\frac{\partial^2\left(E(x)\left(\partial^2 w/\partial x^2\right)\right)}{\partial x^2}+\left[\int_0^L\left(\frac{\partial w}{\partial x}\right)^2\frac{E(x)A}{2L}dx\right]\frac{\partial^2}{\partial x^2}\left[(e_0a)^2\frac{\partial^2 w}{\partial x^2}-w\right]-f(x,t)+(e_0a)^2\frac{\partial^2 f}{\partial x^2}=\rho(x)A\frac{\partial^2}{\partial t^2}\left[(e_0a)^2\frac{\partial^2 w}{\partial x^2}-w\right]+k_w\left[(e_0a)^2\frac{\partial^2 w}{\partial x^2}-w\right]+c_1\frac{\partial}{\partial t}\left[(e_0a)^2\frac{\partial^2 w}{\partial x^2}-w\right]+c_3\frac{\partial}{\partial t}\left[(e_0a)^2\frac{\partial^2 w}{\partial x^2}-w\right]^3+q_w-(e_0a)^2\frac{\partial^2 q_w}{\partial x^2} \tag{2}$$

where $f(x,t)=\int_A\overline{f}_z dz=\xi AH_x^2\frac{\partial^2 w}{\partial x^2}$, ξ is the magnetic field

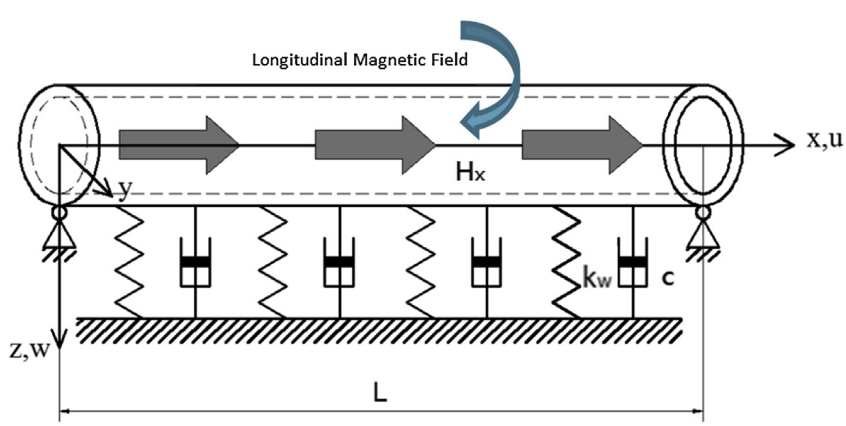

Fig. 1. Single-walled carbon nanotubes embedded in viscous elastic matrix with longitudinal magnetic field.