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Modeling framework for free edge effects in laminates under thermo-mechanical loading

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ABSTRACT

In this paper, a novel Quasi-2D (Q-2D) plane strain formulation for predicting interlaminar stresses in multi-directional laminates is developed and implemented within the finite element method framework. In particular, analyses of free edge stresses in composite laminates subjected to uniform axial and/or thermal loading are conducted. The Q-2D modeling approach presented here is validated by comparing the predicted interlaminar stresses with corresponding 3D models and previously published data. Computational time required for determining the interlaminar stresses using Q-2D model is approximately 30 times lower than the 3D analysis of the same laminate. Also, the Q-2D model is implemented within a commercially available software by modifying a 3D model to behave like a 2D model. This is particularly advantageous over developing an in-house finite element method code to capture the complex free edge stress states in multi-directional composites. Hence, the current framework can potentially be used as a computational tool for efficiently predicting the interlaminar stresses in different laminates subjected to thermo-mechanical loading, which can assist in determining interlaminar regions susceptible to free edge delamination.

1. Introduction

Interlaminar regions in polymer based layered carbon fiber composites are critical regions that are most susceptible to delamination under static and dynamic loading. Debonding or delamination is observed to be a significant failure mechanism in layered composites with considerable visible damage when subjected to load types like edge-wise and through-thickness compression, flexure, and dynamic impact. Delamination type failure is directly influenced by the strength and toughness of interlaminar regions. This is due to significant localized stresses that occur at the interlaminar regions, particularly at the free edges due to mismatch in property between plies, which commonly referred to as the "free edge effect" [1]. Hence, accurate determination of stress distribution near the free edges is very important due to their significant impact on delamination or transverse cracking in layered multi-directional laminates.

Stress state near the free edge is three dimensional in nature and classical lamination theory (CLT) is unable to determine these stresses accurately [1,2]. Therefore, various analytical and numerical approaches such as closed form analytical solutions, boundary layer theories, layer-wise theories, finite difference method and finite element method have been used by earlier researchers to calculate interlaminar stresses near the free edges. Puppo and Evensen [3] proposed the first analytical method to determine interlaminar stresses in a composite laminate. Pagano [4] developed a higher order plate theory to evaluate the interlaminar stresses, and Hsu and Herakovich [5] studied the free edge effects using perturbation and limiting free body approach for angle ply laminates. Tang [6], Davet and Destuynder [7], and Lin and Ko [8] studied similar problems using boundary layer theory. Pipes and Pagano [9] used an approximate elasticity solution, while Pagano [10] and Lekhnitskii and Fern [11] used variational principle to study the free edge effects in laminates. Yin [12,13] used a variational method that utilized Lekhnitskiis stress function for laminates under combined mechanical loading. Later, Yin [14] utilized the principle of complementary energy based on polynomial stress functions to evaluate the interlaminar stresses in laminates subjected non-uniform thermal loading. Andakhshideh and Tahani [15] used three dimensional multi-term extended Kantorovich method to investigate the interlaminar stresses near the free edges

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of general composite laminates under axial and shear loads. Amrutharaj et al. [16] investigated the free edge effects in composite laminates under uniaxial extension using the concept of fracture process zone. D'Ottavio et al. [17] conducted a comparative study of different plate theories for free edge effects in laminated plates and established that Layer-Wise (LW) theories are capable of capturing the free edge effects, while Equivalent Single Layer (ESL) theories fail. Even though the LW theories can capture the edge effects sufficiently well, they are computationally expensive [2] and the results often depend on the number of sublayers considered within each ply [18].

Pipes and Pagano [19] developed the first finite difference based numerical method to solve the two dimensional governing elasticity equations for calculating the interlaminar stresses of long symmetric laminate under uniform axial strain. Later, Atlus et al. [20] and Salamon [21] used three dimensional finite difference method to determine the interlaminar stresses in angle-ply laminates. Wang and Crossman [22,23] investigated edge effects in symmetric composite laminates subjected to uniform axial strain and thermal loading using finite element method. Herakovich et al. [24], Isakson and Levy [25], Rybicky [26], Kim and Hong [27], Icardi and Bertetto [28], Lessard et al. [29] and Yi and Hilton [30] also used finite element method to study free edge effects in laminates. Spilker and Chou [31] used a hybrid stress based finite element method, Lee and Chen [32] used a layer reduction technique, Robbins and Reddy [33] used a displacement based variable kinematic global local finite element method and Gaudenzi et al. [34] used a three dimensional multilayer higher order finite element method to study similar problems. Lorriot et al. [35] investigated the onset of delamination in carbon/epoxy laminates by developing a model based on stress criterion.

From the previous studies mentioned above, it is well established that free edge effects are dominant in multidirectional laminates and result in very high interlaminar stresses that prematurely initiate inter-layer delamination. Therefore, determining interfaces with very large interlaminar stresses is critical for the assessment of delamination driven failure. Interfaces most susceptible to delamination can be determined and strengthened accordingly during manufacturing to reduce their susceptibility to failure. Towards that, a Quasi-2D formulation within the finite element method (FEM) framework is established in this paper for determining delamination prone interlaminar regions in multidirectional laminates subjected to thermo-mechanical loading.

In the current paper, a variational formulation presented by Martin et al. [36] is extended for combined thermal and axial loading on multi-directional laminates, referred to as “Quasi-2D” (Q-2D) formulation. Q-2D formulation is implemented within the FEM framework for accurately determining stress distribution near the free edges for cross-ply ($[0/90]_s$) and quasi-isotropic ($[45/-45/90/0]_s$) laminates. A novel technique is developed in this paper for implementing the Q-2D model within the finite element framework, which involves modifying a thin slice of a 3D laminate to behave like a generalized 2D model by enforcing multi-point constraints. The approach presented in this paper is compared against corresponding 3D models and previously published data for establishing its validity and accuracy.

Other numerical approaches by earlier researchers have investigated the full 3D model to capture the free edge effects. For example, Icardi and Bertetto [28] used a 3D modeling approach with 20 noded quadratic isoparametric brick elements and 15 noded quadratic singular wedge elements for capturing the stress state at the free edges. Lessard et al. [29] also used 3D finite element method with 20 noded quadratic brick elements for the same. Raju and Crews [37] used a combined rectangular and polar mesh for their analysis, where the rectangular mesh was used to determine the stress distributions and polar mesh was used to investigate the stress singularities. But in the current paper, accurate stress states at the free edges are captured using a Quasi-2D reduced model, which takes about 30 times lesser computational time than a 3D model. Another key advantage of the Q-2D model is the easy implementation within any commercially available finite element software as opposed to the requirement of an in-house code.

This paper is divided into the following sections: Section 2 describes the mathematical formulation for the Q-2D, followed by the details of the implementation within the finite element framework in Section 3. Discussion of results from several case studies is presented in Section 4 followed by conclusions.

2. Mathematical formulation

Consider a laminate of length $2L$ and width $2b$, which consists of N layers (thickness h each) as shown in Fig. 1(a). Tensile load is applied on the edges at $\sum_{+L}$ and $\sum_{-L}$ along the x_1 direction, with free edges at $\sum_0$ and $\sum_{2b}$. A temperature change of ΔT occurs uniformly within the laminate.

The stress components are assumed to be independent of x_1 in regions sufficiently far from the loading surface [19,36], such that the displacement field $\{U\}$ can be defined as,

$$\begin{aligned} U_1(x_1,x_2,x_3) &= \tilde{U}_1(x_2,x_3) + \varepsilon_{11}x_1 \\ U_2(x_1,x_2,x_3) &= \tilde{U}_2(x_2,x_3) \\ U_3(x_1,x_2,x_3) &= \tilde{U}_3(x_2,x_3) \end{aligned} \tag{1}$$

where, ε_{11} is the uniform strain applied on the laminate along the x_1 direction. Displacement field $\{U\}$ and corresponding stress field $\{\sigma\}$ adhere to the following equations:

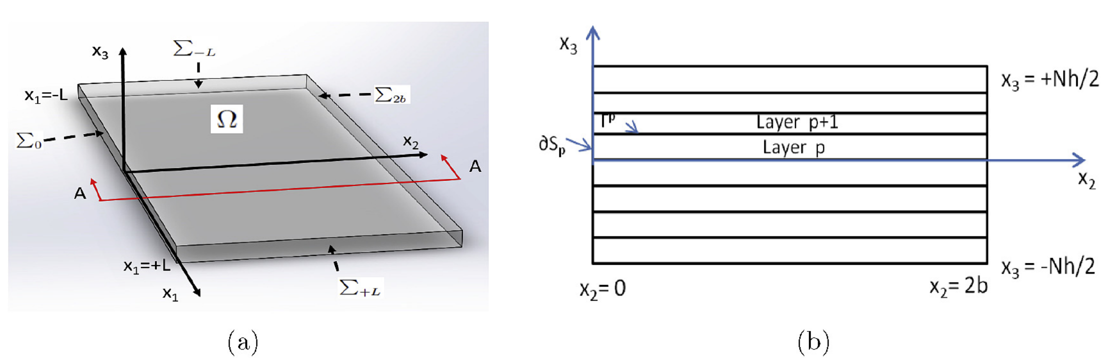

Fig. 1. (a)3D Laminate; (b) Cross-section of a 3D laminate.