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A key-node finite element method and its application to porous materials

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ABSTRACT

Due to the complex microstructures of porous materials, the conventional finite element method is often inefficient when simulating their mechanical responses. In this paper, a key-node finite element method is proposed. First, the concept of key-node is introduced over the element level, and then the governing equations are theoretically derived and corresponding boundary conditions for shape functions of key-node finite element are prescribed. The key-node finite element method is finally established by following the procedure of conventional finite element method to numerically solve the shape functions. Including the information of micro-structures and physical details in shape functions, the key-node finite element is more efficient when preserving a high accuracy, which is validated by typical applications to elastic and elasto-plastic analyses of porous materials. It is straightforward to extend the present method to the three-dimensional case or to solving more challengeable problems such as dynamical responses with high frequencies.

1. Introduction

To effectively model problems with complex micro-structures and/or physical details, much attention has been paid to the modification of the conventional finite element method (CFEM) from many aspects in recent decades. Typical work includes the numerical manifold method (NMM) proposed by Shi in 1991 [1], the generalized finite element method (GFEM) proposed by Babuska et al. in 1996 [2], and the extended finite element method (XFEM) proposed by Moes et al. in 1999 [3]. Although shape functions are enriched in these methods for different purposes, polynomial terms are still essential, whichever are expressed in terms of global area coordinates (e.g. for triangular elements) or local parent coordinates (e.g. for quadrilateral elements) [4,5]. Since the shape functions are a priori given and independent of problems to be solved for, the solution accuracy and efficiency are often case dependent.

To change this situation, Babuska et al. [6] proposed the idea that the basis functions adapted to the specific problem, and then applied it to the problem with rough coefficients. This idea was later developed by Hou et al. [7] to be the multi-scale finite element method (MsFEM), and then further to be the generalized MsFEM (GMsFEM) to perform multiscale simulations for problems without scale separation over a complex input space [8]. Recently, the MsFEM was extended to modeling plane elasticity problems [9,10]. Nevertheless, it is still difficult for the MsFEM to model porous materials because of the complex compatibility between elements.

To this end, the key-node finite element method (KN-FEM) is proposed in this paper. In Section 2, the concept of key node is first presented, and then the interpolation of the KN-FEM is introduced for plane elasticity problems. The governing equations are finally derived to shape functions of the KN-FEM. In Section 3, the corresponding boundary conditions (BCs) are firstly discussed for different key node finite elements, and then shape functions are calculated as compared with those of the conventional finite element if any. Finally, the partition of unity property is verified as well in this section. In Section 4, the KN-FEM is applied to some typical examples with porous materials to validate its accuracy and efficiency. The concluding remarks are made in Section 5.

2. Basic theories

2.1. Concept of key-node finite element

As shown in Fig. 1, for a quadrilateral 4-node element, four nodes are necessary to define the element shape. So it is quite natural to call the four corner nodes the key nodes of quadrilateral element.

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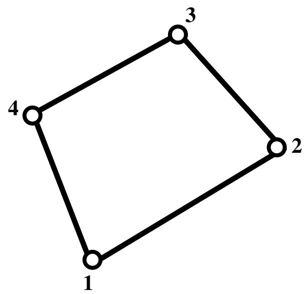


Fig. 1. A quadrilateral element and its four key nodes.

The concept of key node is the generalization of corner nodes. As shown in Fig. 2(a), for the purpose to better interpolation in a quadrilateral element, it is necessary to introduce one more node along edge 1–2. At this point, node 5 is also called a key node, and therefore the element is termed as a quadrilateral element with 5 key nodes. In this fashion, a quadrilateral element with more than 5 key nodes can be defined and then used when necessary. For instance, in Fig. 2(b) is shown a quadrilateral element with 8 key nodes.

The elements shown in Figs. 1 and 2 are all called key-node finite element. It should be noted that key nodes can be at discretion allocated along element edges, no matter what the number is or where the positions are.

2.2. Interpolation of key-node finite element method

In this paper, our attention is confined to plane elasticity problems. In this case, the governing differential equations in stress form are

$$\sigma_{ij,j} + f_i = 0 \quad \text{in } \Omega \tag{1}$$

where σ_{ij} are the components of stress tensor and $f_i(x, y)$ are the components of body force vector. The subscript, j denotes the partial differentiation with respect to coordinate x if $j = 1$ or y if $j = 2$. In addition, as a well-defined problem, over the whole boundary $\Gamma = \partial\Omega = \Gamma_u \cup \Gamma_t$, the essential BCs and natural BCs are prescribed on Γ_u and Γ_t, respectively.

After invoking Hooke's law and the strain–displacement relations of small deformation, in terms of displacement field (u, v), Eq. (1) is rewritten as the following Navier equation [11]

$$\begin{cases} \nabla \cdot (\mu \nabla u) + \dfrac{\partial}{\partial x}[(\lambda + \mu)\theta] + f_x = 0 \\ \nabla \cdot (\mu \nabla v) + \dfrac{\partial}{\partial y}[(\lambda + \mu)\theta] + f_y = 0 \end{cases} \tag{2}$$

where λ and μ are Lame constants. Here, we assume that the material is isotropic but may be inhomogeneous and therefore the two constants vary with (x, y). $\theta = \partial u/\partial x + \partial v/\partial y$ is the displacement divergence in the two-dimensional setting.

For a linear problem, due to the superposition principle [11,12] and considering the basis feature of shape functions in the finite element method, the homogeneous form of Eq. (2) is fundamental, i.e.

$$\begin{cases} \nabla \cdot (\mu \nabla u) + \dfrac{\partial}{\partial x}[(\lambda + \mu)\theta] = 0 \\ \nabla \cdot (\mu \nabla v) + \dfrac{\partial}{\partial y}[(\lambda + \mu)\theta] = 0. \end{cases} \tag{3}$$

To get a closed-form of shape functions, the interpolation of key-node finite element should be [10]

$$\begin{cases} u = \sum \phi^i_{uu} u_i + \sum \phi^i_{uv} v_i \\ v = \sum \phi^i_{vu} u_i + \sum \phi^i_{vv} v_i \end{cases} \tag{4}$$

where ϕ^i_{uu}, ϕ^i_{uv}, ϕ^i_{vu} and ϕ^i_{vv} are shape functions of key node element at node i. Different from the conventional interpolations, coupling terms ϕ^i_{vu} and ϕ^i_{uv} appear in Eq. (4) to reflect the mutual influence of nodal displacements u_i and v_i.

Considering the versatility of the finite element method in approximating possible variations, as a sufficient condition, on substituting Eq. (4) in Eq. (3), we eventually obtain governing equations for the shape functions over each element, i.e.

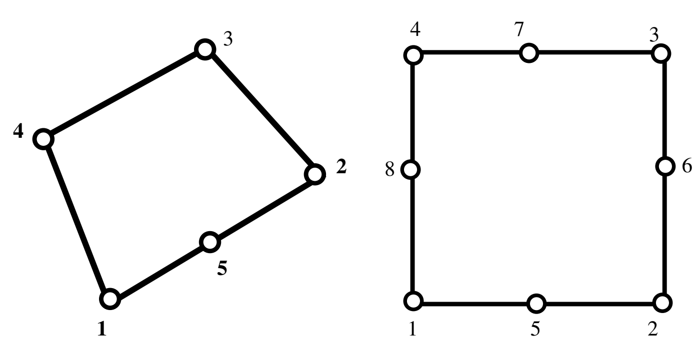
$$\begin{cases} \nabla \cdot (\mu \nabla \phi^i_{uu}) + \dfrac{\partial}{\partial x}\left[(\lambda + \mu)\left(\dfrac{\partial \phi^i_{uu}}{\partial x} + \dfrac{\partial \phi^i_{vu}}{\partial y}\right)\right] = 0 \\ \nabla \cdot (\mu \nabla \phi^i_{vu}) + \dfrac{\partial}{\partial y}\left[(\lambda + \mu)\left(\dfrac{\partial \phi^i_{uu}}{\partial x} + \dfrac{\partial \phi^i_{vu}}{\partial y}\right)\right] = 0 \end{cases} \tag{5a}$$

and

$$\begin{cases} \nabla \cdot (\mu \nabla \phi^i_{uv}) + \dfrac{\partial}{\partial x}\left[(\lambda + \mu)\left(\dfrac{\partial \phi^i_{uv}}{\partial x} + \dfrac{\partial \phi^i_{vv}}{\partial y}\right)\right] = 0 \\ \nabla \cdot (\mu \nabla \phi^i_{vv}) + \dfrac{\partial}{\partial y}\left[(\lambda + \mu)\left(\dfrac{\partial \phi^i_{uv}}{\partial x} + \dfrac{\partial \phi^i_{vv}}{\partial y}\right)\right] = 0. \end{cases} \tag{5b}$$

3. Calculation of the shape functions

It turns out that the shape functions are crucial to the key node finite element. Unfortunately, because the key nodes are often allocated according to complex micro-structures or physical details, it is hard to obtain the analytic solution to Eq. (5) except for very special cases (e.g. see [13]). To this end, a numerical solution via the CFEM on element level is preferred. Such a choice is in reality straightforward because the finite element code can be readily invoked when considering a strong resemblance between Eqs. (5) and (2), only with the specific BCs instead.



(a) With 5 key nodes. (b) With 8 key nodes.

Fig. 2. A quadrilateral element with more than 4 key nodes.