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Co-rotational shell element for numerical analysis of laminated piezoelectric composite structures

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A B S T R A C T

Laminated composite structures consisting of load-carrying and multifunctional materials represent a rather powerful material system. The passive, load-carrying layers can be made of isotropic material or fiber-reinforced composites, while piezoelectric materials represent the most common choice of multifunctional materials for active layers. The multifunctionality of piezoelectric layers is provided by their inherent property to couple mechanical and electric fields. The property can thus be used to sense deformations or produce actuating forces. A highly efficient 3-node shell element is developed for modeling piezoelectric laminated composite shells. The equivalent single-layer approach and Mindlin-Reissner kinematics are used in the element formulation together with the discrete shear gap (DSG) technique to resolve the shear locking and strain smoothing technique to improve the performance. Piezoelectric layers are assumed to be polarized in the thickness direction thus coupling the in-plane strains with the electric field oriented in the thickness direction. The co-rotational FE formulation is used to account for geometrically nonlinear effects. Numerical examples cover linear and geometrically nonlinear static and dynamic cases with piezoelectric layers used as actuators and sensors.

1. Introduction

Thin-walled structures render some 80% of all engineering structures and are still a growing portion of engineering structures in a broad range of sizes and quite diverse applications. They are a result of the tendency to reduce the structural dead-load but keep at the same time the high level of carrying capacity and stiffness. It is the combination of the shape and thinness of the walls that provides those advantages. The advantages are further improved by use of modern engineering materials - laminates, with layers made of various materials that could be isotropic or, more frequently, fiber-reinforced composites. Orthotropic fiber-reinforced composite laminates offer vast options for tailoring material properties through the choice of constituent materials, fiber orientation, number, thickness and sequence of layers.

Despite all these advantages, composite laminates may also suffer from structural stability issues and are sensitive to vibrations. The idea behind the term *smart/adaptive structures* offers a great potential to cope with such challenges. The term has been adapted by the engineering community two decades ago to redefine the concept of structures from a conventional passive deformable system to an active controllable system with inherent self-sensing, diagnosis, actuation and control capabilities [1]. The use of multifunctional materials enables application of active elements (sensors and actuators) with excellent capability of structural integration. Piezoelectric materials represent quite a common choice of multifunctional materials for the considered type of structures, which is due to their operational frequency range as well as stroke and force range they can produce, when shaped for the use with thin-walled structures. Their inherent property to couple mechanical and electric fields is used for this purpose. Since it is a two-way coupling, it can be used for actuation by producing desired forces through a predefined electric potential (reverse piezoelectric effect), and for sensing, as deformations give rise to a strain-proportional electric field (direct piezoelectric effect). Such systems have a broad range of applicability, including vibration suppression [2–4], structural health monitoring [5,6], shape control [7,8], to name but a few.

Successful design of piezoelectric laminates and appropriate control laws calls for efficient and reliable approaches for modeling

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and simulation of their behavior. Whereas some researchers provided analytical considerations of piezoelectric laminated structures [9–11], the attention was mainly turned to the finite element method (FEM) as a predominant numerical method in the field of structural analysis. Numerous developments of piezoelectric beam, plate and shell elements are the best prove of how enticing this research field is. An exhaustive overview would be prohibitively long. An interested reader may address the survey from Benjeddou [12] for a thorough overview of the development in the field during the '90s and the development continued at the same pace in the years to come. Although it was confined to the considered type of material system and structures, still a few major streams of development can be distinguished. Some of the developments were aimed at high fidelity solid elements, with various techniques used to improve the performance. This includes the mixed variational formulation applied with an 8-node piezoelectric solid shell element by Klinkel and Wagner [13], and the assumed strain technique applied with an 18-node element by Lee et al. [14]. Willberg and Gabbert [15] applied the isogeometric approach to develop a 3D piezoelectric finite element for smart structures. Li at al [16]. proposed 2D and 3D elements with the smoothed strain technique for piezoelectric structures.

If the global structural behavior is aimed at, 2D elements, i.e. plates and shells, offer greater numerical efficiency. A large number of developed elements use the equivalent single-layer approach and are mainly based either on the classical laminate theory that implements Kirchoff-Love kinematics (e.g. Refs. [17,18]), or the first-order shear deformation (FSDT) theory with Mindlin-Reissner kinematics. The latter was more frequently used in the FEM developments as it includes the transverse shear effects and requires the C^0-continuity from the shape functions (compared to the C^1-continuity needed for the classical laminate theory). The developments cover broad range of finite elements including linear triangular [19] and quadrilateral [20] shell elements, biquadratic 8-node [21] and 9-node [22] shell elements, etc. As shell elements are notorious for the shear and membrane locking phenomena, various techniques, such as discrete shear gap (DSG) [20], mixed-interpolation of tensorial components (MITC) [23], selectively [21] and uniformly [22] reduced integration, etc. Were applied to alleviate the problem. The developed elements were used to investigate further effects in modeling electro-mechanical coupled field, such as the convergence behavior of FEM results [24], and, for users' convenience, some developments were also implemented in commercial FEM programs [25]. The isogeometric approach was also considered in the development of 2D elements for piezoelectric laminates. Phung-Van et al. [26] used it in combination with a higher-order shear deformation theory.

Layerweise theories were also addressed to provide finite elements that stand between the 2D elements based on the equivalent single-layer approach and 3D elements, regarding the numerical effort and achieved accuracy. For this purpose, the Carrera Unified Formulation (CUF) for multilayered plates and shells [27] is frequently applied. Based on it, Cinefra et al. [28] developed a 9-node plate element for static analysis using the MITC technique and variable through-the-thickness layer-wise kinematics. This development was later extended to cover free-vibration analyses of piezoelectric plates [29]. Milazzo [30] used the approach that reduces the coupled-field problem to mechanical one and implemented both equivalent single-layer and layer-wise approaches.

Geometric nonlinearities were significantly less addressed in the available literature and this is one of the contributions this paper aims at. A linear triangular shell element, whose mechanical part is based on the development by Bletzinger et al. [31] and Nguyen-Thoi et al. [32], was extended by the authors of this article to include piezoelectric layers polarized in the thickness direction and to cover geometric nonlinearities characterized by finite local rotations but small strains. The co-rotational (CR) FEM formulation [33–35] is used for the purpose. Application of the element for static and dynamic actuator and sensor cases will be demonstrated.

2. 3-Node piezoelectric shell element

The choice to develop the linear triangular shell element was motivated by its high numerical efficiency and meshing ability. However, those advantages are accompanied by the disadvantage of relatively stiff element behavior. Since it is a flat element, shell behavior is obtained by directly superposing the plate and membrane behavior. The mechanical field of the element relies on the development by Bletzinger et al. [31] and the DSG technique is used to alleviate the shear locking. Nguyen-Thoi et al. [32] used the strain smoothing technique to further improve this element, i.e. to avoid large strain and stress oscillations between adjacent elements and to render the element formulation independent from node numbering. Another aspect that talks in favor of the linear triangular element is the objective of its implementation into the co-rotational FEM formulation for geometrically nonlinear analysis. Since the rigid-body rotation is considered element-wise (one rotation matrix per element), finer meshes are needed for adequate accuracy regardless of the element properties. In what follows, only the basic equations that describe the element mechanical and electric fields are given.

2.1. Element geometry and mechanical field

Two coordinate systems are used in the element formulation – the global (x, y, z) and local (x′, y′, z′) coordinate systems. The local coordinate system is essential for the description of element geometry, implementation of kinematics and constitutive equations but also for the description of the electric field and piezoelectric coupling. It is defined so as to have one of its axes, the x′-axis, oriented from element node 1 towards node 2, while the z′-axis is perpendicular to the element surface, Fig. 1. It takes very basic vector algebra to define the unit vectors of the local coordinate system, $\{e_{x'}\}$, $\{e_{y'}\}$ and $\{e_{z'}\}$ and this is omitted here for the sake of brevity.

The element uses linear shape functions which are defined in a manner common for triangular elements. Any point within the element, with local coordinates x′ and y′, forms 3 sub-triangles in the element. The shape function of node i at any point (x', y') in the element domain is defined as a ratio of the corresponding sub-triangle surface area (defined by the point and the remaining two element nodes) and the element surface area. Hence, the shape functions for all 3 element nodes read:

$$
\begin{aligned}
N_1(x',y') &= \frac{1}{2A_e}[(x'_1y'_3 - x'_3y'_2) + (y'_2 - y'_3)x' + (x'_3 - x'_2)y'] \\
N_2(x',y') &= \frac{1}{2A_e}[(x'_3y'_1 - x'_1y'_3) + (y'_3 - y'_1)x' + (x'_1 - x'_3)y'] \\
N_3(x',y') &= \frac{1}{2A_e}[(x'_1y'_2 - x'_2y'_1) + (y'_1 - y'_2)x' + (x'_2 - x'_1)y']
\end{aligned}
\tag{1}
$$

with x'_i and y'_i, $i = 1,2,3$ denoting the local coordinates of the element nodes, while A_e is the element surface area. The shell thickness is assumed to be perpendicular to the mid-surface and the element geometry with respect to the local coordinate system is given as: