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On the minimal mass design of composite membranes

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ABSTRACT

This paper will show that a simple membrane (e.g. balloon) is not always the minimal mass pressure vessel. Of course one could always add cables to support the membrane, or plate, to reduce the thickness and hence mass required of the membrane. By minimizing the sum of the mass of the membrane plus the mass of the cable network, a minimal mass solution for the pressure vessel can be found. We will show the necessary and sufficient conditions (involving material choices and cable network topology) for which a composite system, composed of a membrane and cable network, has less mass than a membrane alone. The main motivator for this study is a spin-stabilized pressurized space structure useful for artificial gravity habitats.

To contain and support the membrane, we will optimize the topology of the class of prestressable structures called tensegrity. These structures are usually composed of a network of axially-loaded compressive and tensile members. In the pressurized examples of this paper, inflation provides the compressive forces, and hence the optimized tensegrity topology eliminates the compressive structures when they are not needed. The minimal mass designs herein produce easily-tunable prestressed networks, and control systems that allow deployment and stiffness tuning features.

The minimal mass design of those structures, for different load combinations (pressure loads and centrifugal loads), produces a composite structure composed of membranes and cable network. Two different composite systems are analyzed and compared with simple uniform membranes: the first system is composed of a cylindrical membrane supported by circular ring cables, the second system is composed by a membrane surrounded by ring cables and longitudinal cables. The comparison of the minimal mass designs with the single continuous membrane clearly highlights the advantage of the composite systems made of high-performance materials. When the material of the membrane and cables are the same, there is no advantage to the composite system, but various requirements usually demand different properties of the membrane and cable material.

1. Introduction

A strong motivator for this paper is the need to build light-weight structures in space. Early efforts of space exploration built the spacecraft on earth and launched it to space at great launch costs. The future of space exploration will save launch costs by building the structures in space, launching only raw materials, greatly reducing the volume of the required launch. Perhaps the most critical innovations needed in space, and needs that must be fulfilled before humans can occupy space for long terms, is gravity and shielding. Both of these challenges require light-weight construction of spinning habitats. No research is available to quantify the level of gravity that is required for long term health. Habitats on the Moon or Mars can provide less than 1/3 g, and we do not know how children's bone would grow in such small g level. The only way to establish a scientific quantification of the healthy g level required is to build a variable g facility in space, using centrifugal forces. Hence the requirement of mass efficiency for space structures and the need for centrifugal forces applied to the structures motivate this paper.

Structural optimization has a long history, but, loosely speaking, the methods can be characterized by three approaches; FEM, Gridding, and Fractals [1–3]. Finite Element Methods (FEMs) start with mass everywhere in the allowed space (e.g. a brick) and iteratively delete individual finite elements (usually guided by gradient techniques). Gridding techniques, as in Ref. [4]; make the initial structure less dense by starting with a finite number of

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connected nodes, and then delete connections one at a time to minimize mass. Fractals fill space in a self-similar fashion, and the optimal number of self similar iterations can be based upon some artistic appeal. But adding guaranteed mechanical properties to fractal theory is relatively new, starting with the tensegrity work [5–7].

Fractal work on tensegrity concepts have proposed another method for structural optimization, filling the volume with a self-similar repetition of the chosen tensegrity element, until mass is minimized. The chosen tensegrity element itself is based upon an optimization problem, using a fundamental fractal object that has some optimal mechanical property, such as compressive strength. The number of self-similar iterations is increased until the total mass required to support the given boundary conditions is minimized. This gives an optimal complexity (to yield minimal mass) of the fractal network. The efficiency of the method depends upon the choice of the particular tensegrity topology in the fractal, but in each case the method produces an optimal complexity of the fractal.

In many cases, the best solution for minimal mass is composed of *tensegrity* structures [5]. Such structures are a stable network of sticks (compressive elements) and strings (tensile elements), that can be easily controlled, since the dynamic models of such fundamental elements (axially-loaded rods and strings) are extremely reliable. The complete nonlinear dynamics of any tensegrity system composed of massless cables and rigid rods are given in a series of papers [8–14]. The *tensegrity* choice is motivated to ensure both minimal mass solutions [6,7,15–19], and deployability [20,21]. *Meta-materials* have also been constructed with amazing electrical or acoustic properties [22–27]. Recent contributions have shown the potential applications of mechanical metamaterials to seismic isolation [28], as well as the use of tensegrity concepts for the optimal design of composite reinforcements for existing masonry structures [29]. Other papers show amazing mechanical properties and *composite* systems [30,31].

It is commonly accepted that continuous membranes like balloons, cylinders and their combinations are optimal minimal mass structures to carry an internal outward pressure [32]. We proove herein that the mass of such continuous structures can be further reduced by making composite structures of a cable network and membrane. The efficiency of such approach is validated by numerical results that highlight significant mass reduction of the composite structures compared to the original continuous structure. Even if single continuous membranes are commonly accepted as minimal mass systems, appropriate material choice is crucial to further mass reductions. We show that the use of those materials merge perfectly with the choice of *tensegrity* structures helping continuous membranes.

Previous ideas on space habitats can be found in Refs. [33–42].

The next generation of space habitat must also take into account *shielding* structures that will protect from the unhealthy space radiation [43–45]. The design of such shielding technologies is still an open issue, but the mass needed can be added to the material of the membrane, as suggested herein. Moreover, the space habitats require light, even though the radiation shield prevents any direct sunlight. The sunlight might be reflected by mirrors to the interior. Hence the walls of the habitat should be semi-transparent to transmit enough light. Hence, we use membrane material that transmits some light, such as UHMWPE (Ultra High Molecular Weight Polyethelene).

This paper optimizes the cable topology and minimizes the total mass of the membrane and the cables. Other optimization studies of cylindrical shells include [46]; who studied composite cylindrical shells with Finite Element Method [47]; who conducts an experiment on the energy absorption of composite tubes; and [48] who obtained the frequencies of vibrations of rotating carbon nanotube cylinders.

Deniz et al. [49] also studied the vibration modes of conical shells resting on elastic foundations and made of composite materials employable as thermal barriers. Jing et al. [50] analyzed cylindrical shells subject to air blast loading made of sandwich cells with a foam core. Bilston et al. [51] conduct an experimental campaign on composite tubes made of concentric circular tubes with an inner structural foam and subject to quasi static and dynamic loads.

The major contribution of this paper is the analytical relationships useful for the design of inflatable and expandable structures with parametric geometry. These approaches allow multi-scale topologies that are suitable for optimization under specified constraints, such as mass and/or stiffness minimization. This paper includes both centrifugal forces from spinning, and internal pressure. Obviously the shape of the vessel is constrained by stability constraints for the spinning structure. This paper shows that composite systems made of tensile networks and membranes can reduce the mass compared to just one continuous membrane. We have shown how to reduce the mass by adding different kind of cables to the original membrane structure. The resulting structure designs have application in composite tubes/cylinders, tanks and containers, or even civil underground structures like tunnels, wells and domes.

The remainder of the paper is organized as follows. In Sec 2 we study the equilibrium of a network of spinning cables (straight and circular). Sec 3 describes the parametric topology and the loads of cylinders made of cables and membranes. Secs 4 and 5 study two composite systems in which the total mass can be less then the mass of a single membrane thanks to the addition of a net of cables. Then, the two composite systems are directly compared in Sec 6. Conclusions and future work are reported in Sec 7.

2. Design of spinning pressurized hulls

In this section we consider the minimal mass hull design composed of a single membrane, subjects to internal pressure and spinning motion about the longitudinal axis. Let us consider a thin walled cylinder with internal radius r, wall thickness Δ, and total length L (Fig. 1(a–b)). The cylinder is subjected to an internal pressure P_0 and it is spinning about its longitudinal axis with a constant angular velocity ω. The membrane is made of a material with mass density ρ and yielding strength σ. The cylinder also carries an internal payload material with mass density ρ_p, and thickness Δ_p.

Theorem 2.1. *The minimum required mass and thickness of the membrane described in Fig. 1 are*:

$$m = \mu\, m_0, \quad \Delta = \mu\, \Delta_0 \tag{1}$$

where:

$$m_0 = 2\pi r \Delta_0 \rho L, \quad \Delta_0 = \frac{P_0 r}{\sigma} + \frac{\rho_p \Delta_p r_p r \omega^2}{\sigma}, \quad \mu = \frac{1}{1 - \frac{\rho}{\sigma} r^2 \omega^2} \tag{2}$$

It is worth noting that the quantity m_0 in (1) assumes the meaning of a *rest mass*, and the quantities m and μ recall *relativistic* mass and *Lorenz* factor respectively [52].

Proof. The mass of the cylinder and the payloads are:

$$m = \rho(2\pi r)\Delta L, \quad m_p = \rho_p(2\pi r_p)\Delta_p L \tag{3}$$

The centrifugal forces of the cylinder and the payloads are: