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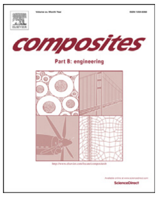

Optimal design of low-frequency band gaps in anti-tetrachiral lattice meta-materials

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ABSTRACT

The elastic wave propagation is investigated in a beam lattice material characterized by a square periodic cell with anti-tetrachiral microstructure. With reference to the Floquet-Bloch spectrum, focus is made on the band structure enrichments and modifications which can be achieved by equipping the cellular microstructure with tunable local resonators. By virtue of its composite mechanical nature, the so-built inertial meta-material gains enhanced capacities of passive frequency-band filtering. Indeed the number, placement and properties of the inertial resonators can be designed to open, shift and enlarge the band gaps between one or more pairs of consecutive branches in the frequency spectrum. In order to improve the meta-material performance, several nonlinear optimization problems are formulated. The largest among the band gap amplitudes in the low-frequency range is selected as suited objective function. Proper inequality constraints are introduced to restrict the admissible solutions within a compact set of mechanical and geometric parameters, including only physically realistic properties of both the lattice and the resonators. The optimization problems related to full and partial band gaps are solved by using a globally convergent version of the numerical method of moving asymptotes, combined with a quasi-Monte Carlo multi-start technique. The optimal solutions are numerically computed, discussed and compared from the qualitative and quantitative viewpoints, bringing to light the limits and potential of the meta-material performance. The clearest trends emerging from the numerical analyses are pointed out and interpreted from the physical viewpoint. Finally, some specific recommendations about the microstructural design of the meta-material are synthesized.

1. Introduction

Composite lattice structures are encountering an increasing success in a number of advanced applications within both established and emerging engineering fields. If the mechanical virtues of lattice structures can traditionally be attributed to their optimal material usage and their high designable properties, a growing research attention is currently being focused on their unconventional functional performances. Indeed, lattice materials and meta-materials may offer natural and unique attitudes to develop valuable and adaptable capacities of spatial morphing, remote sensing, health monitoring, active damping, energy harvesting.

Traditional structural realizations of spatial beam lattices include tetrahedral trusses, double-curvature cable networks and hexagonal honeycombs [1–3]. Starting from this well-defined background, a deepening interest has been attracted in the last decades by the geometrical and mechanical design of periodic lattices, with challenging perspectives towards the employment of multi-scale hierarchical schemes and the achievement of multi-physical functionalities [4–9]. As major findings of this optimization trend, novel promising architectural *topologies* have been discovered and new challenging structural *typologies* have been proposed. Among the novel geometric topologies – for instance – the cellular layouts based on chiral and anti-chiral cellular symmetries have demonstrated distinctive elastic properties, such as a marked auxeticity and shear rigidity, together with outstanding capacities of fracture toughness, indentation resistance and energy absorption [10–20]. Among the new structural typologies – for instance – the tri-dimensional tendon-strut systems based on the tensegrity concept have virtuously conjugated remarkable strength-to-lightness ratios with extreme properties of softening/hardening elasticity and high degrees of spatial deployability

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[21–25].

Chiral cellular materials have also drawn many research efforts specifically focused on their high design flexibility as fully mechanical filters for the transmission and dispersion of elastic waves [26–28]. Indeed, the chiral microstructures, characterized by a composite pattern of rings and ligaments, are suited for fine-grained customizations of the acoustic material properties. Employing the Floquet-Bloch theory for periodic media [29–32], the band structure properties of different chiral materials have been extensively studied, with reference to beam lattice microstructural models [26,27,33] as well as to equivalent homogenized continua [28,34]. Within this framework, enhanced capabilities of frequency band filtering can be achieved by the *inertial meta-materials*, which leverage the negative effective mass density that can be obtained with the introduction of intra-ring massive disks [35–37]. By virtue of the ring-disk elastic coupling, these auxiliary oscillators work as local inertial resonators which – if properly tuned – can open, shift and enlarge the spectrum band gaps in response to specific design requirements [38–42].

In the rich library of chiral topologies, the class of anti-tetrachiral materials, first identified as the structural solution of a topological optimization problem [43], is highly attractive for its strong auxeticity, accompanied by a marked anisotropy of the elasto-dynamic response [11,13,34,44–46]. In terms of dynamic analyses, the micro-structural complexity of the anti-tetrachiral periodic cell requires several independent parameters for a minimal but complete description of the elasto-dynamic material behavior. Furthermore, the tetra-atomic cellular configuration implies a quadruple number of Lagrangian coordinates with respect to the mono-atomic layout of the trichiral, tetrachiral or hexachiral materials. Consequently, the band structure of the material possesses a high spectral density, with several dispersion curves interacting to each other in the same frequency range, as well as a strong sensitivity of the spectral properties to a number of structural parameters. To date, the wave propagation spectrum of the anti-tetrachiral material has been determined by solving the eigenproblem governing the Floquet-Bloch theory through numerical continuation methods or asymptotic perturbation techniques [33,34]. Some preliminary parametric analyses have also confirmed the possibility to control the band structures by means of tuned inertial resonators [47]. Nonetheless, the high spectral density and the large variety of design parameters make the anti-tetrachiral meta-material a challenging benchmark for the application of structural optimization strategies oriented to improve and maximize its performance as a mechanical filter.

Based on these motivations, the present paper focuses on the spectral optimization of the anti-tetrachiral periodic meta-material, with the objective to maximize the largest amplitude among the low-frequency band gaps in the Floquet-Bloch spectrum. Fixed the anti-tetrachiral topology, the optimization concerns the key parameters describing both the microstructural model of the periodic cell and the elasto-dynamic properties of the inertial resonators. The number and placement of the resonators are accounted for as complementary optimization variables. The paper is organized as follows. The physical-mathematical description of the periodic meta-material is formulated in the framework of linear elasticity (Section 2). In particular, a beam lattice model is defined to govern the free undamped vibration of the periodic cell (Section 2.1). Therefore, the Floquet-Bloch theory is invoked to describe the wave propagation, and the significant spectral properties are expressed as functions of a physically admissible set of mechanical parameters (Section 2.2). The optimization problems are outlined (Section 3), and then formally stated with proper distinction between the maximization of full and partial band gaps (Section 3.1). After a brief introduction to the numerical solution methods, which are featured by global convergence and quasi-random initialization (Section 3.2), two scenarios or levels of optimization problems are tackled. In the first scenario, all the possible cell configurations, which differ to each other for the number and placement of the resonators, are independently analyzed and optimized (Section 3.3). In the second scenario, the best cell configuration is fixed and a further optimization process is focused only on the resonator properties (Section 3.4). The optimization results are discussed from a qualitative and quantitative viewpoint, some design recommendations are pointed out on the base of the clearest trends emerged from the analyses and, finally, concluding remarks are offered.

2. Anti-tetrachiral material vs meta-material

A composite periodic material, characterized by square cells, is considered to realize a regular tiling of the infinite bi-dimensional domain (Fig. 1a). The geometric and mechanical properties of the generic cell are featured by a double symmetry, according to an anti-tetrachiral planar topology (Fig. 1b). The cell microstructure, with characteristic size ε, is composed by four circular rings connected by twelve tangent ligaments. The *rolling-up* mechanism, responsible for the auxetic behavior, consists in the opposite-sign, iso-amplitude rotations developed by any pair of adjacent rings in-a-row (or column), when the cell is stretched along one of the symmetry axes. With respect to the traditional anti-tetrachiral material, the introduction of intra-ring (mass-in-mass) resonators can give birth to a high-performing inertial meta-material, characterized by enhanced capacities of filtering low-frequency bands of elastic harmonic waves.

2.1. Beam lattice model

A beam lattice model is formulated to describe the linear elasto-dynamic response of the cellular composite with unit thickness. A rigid body model is assumed for the massive and highly-stiff rings, possessing identical mean radius R. The ring centers are located at the four corners of an ideal internal square, concentric with the external cell boundary. The ring width W is considered a free parameter, allowing the independent assignment of the rigid body mass M and moment of inertia J, fixed the material density ϱ. A linear, extensible, unshearable model of massless beam is employed for all the light and flexible ligaments, in the small-deformation range. As long as the beam-ring connections nominally realize perfectly-rigid joints, the natural length L of the *inner* horizontal and vertical ligaments coincides with half the side of the square cell. By virtue of the periodicity, the cell boundary crosses the midspan – and halves the natural length – of all the *outer* ligaments. Assuming a linear elastic material (with Young's modulus E_s) and a rectangular cross-section shape (with area A and second area moment I depending on the width w_s) for each ligament, all the beams have identical extensional E_sA and flexural rigidity E_sI. The effects of a homogeneous soft matrix, which may likely embed the microstructure [34], are neglected as first approximation.

Moving from this structural layout, a meta-material can be realized by supplying each ring with a light soft annular filler, hosting a central heavy circular inclusion, serving as inertial resonator with adjustable mechanical properties. All the inclusions are assumed identical and highly stiff, so that they can be modelled as rigid disks, co-centered with the respective housing rings, with body mass M_r and moment of inertia J_r depending on the radius R_r and the material density ϱ_r. As long as the internal coupling provided by the filler can be assumed linearly elastic (with Young's modulus E_r and Poisson ratio ν_r), the ring-resonator differential

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